Forecasting VaR in Spot and Futures Equity Markets

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Abstract

The goal of this paper is threefold. First, we present evidence for the validity of the ARMA-GARCH model with tempered stable innovations to estimate one-day-ahead VaR in the cash and futures markets for three stock indices — S&P 500, DAX 30, and Nikkei 225 — for the period December 14, 2004 to December 31, 2008. This is the first time that testing of this model has been done for equity futures. Second, based on the vast theoretical and empirical literature suggesting its strong link with volatility, we test for the first time whether adding trading volume to the classical tempered stable model improves the forecasting ability of the model. Finally, we compare the number of times that the market data drop below the corresponding one-day-ahead VaR estimations for both spot and futures equity markets in CTS with and without models including trading volume.

JEL classification: G12; G14; G17

Keywords: Tempered stable distribution; Value-at-risk; Stock index futures contracts; Trading volume

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1. Introduction

Predicting future financial market volatility is crucial for risk management of financial institutions. The empirical evidence suggests that a suitable market risk model must be capable of handling the idiosyncratic features of volatility, that is, daily returns time variant amplitude and volatility clustering. There is a well-developed literature in financial econometrics that demonstrates how autoregressive conditional heteroskedastic (ARCH) and generalized ARCH (GARCH) models — developed by Engle (1982) and Bollerslev (1986), respectively — can be employed to explain the clustering effect of volatility. Moreover, the selected model should consider the stylized fact that asset return distributions are not normally distributed, but instead have been shown to exhibit patterns of leptokurtosis and skewness.

Taking a different tact than the ARCH/GARCH approach for dealing with the idiosyncratic features of volatility, Kim et al. (2009) formulate an alternative model based on subclasses of the infinitely divisible (ID) distributions. More specifically, for the S&P 500 return, they empirically investigate five subclasses of the ID distribution, comparing their results to that obtained using GARCH models based on innovations that are assumed to follow a normal distribution (what we refer to as simply normal innovations). They conclude that, due to their failure to focus on the distribution in the tails, GARCH models based on the normal innovations may not be as well suited as ID models for predicting financial crashes.

Because of its popularity, most empirical studies have examined value at risk (VaR) as a risk measure. These studies have focused on stock indices. For example, Kim et al. (2010), Sun et al. (2010), and Asai and McAleer (2009) examine the S&P 500, DAX 30, and Nikkei 225 stock indices, respectively. A few researchers have studied this risk measure for stock index futures contracts: Huang and Lin (2004) (Taiwan stock index futures) and Tang and Shieh (2006) (S&P 500, Nasdaq 100, and Dow Jones stock index futures). As far as we know, there are no empirical studies comparing VaR spot and futures indices. For this reason, we compare the predictive performance of one-day-ahead VaR forecasts in these two markets.

We then introduce trading volume into the model, particularly, within the GARCH framework. There are several studies that relate trading volume and market volatility for equities and equity futures markets. Studies by Epps and Epps (1976), Smirlock and Starks (1985), and Schwert (1989) document a positive relation between volume and market volatility. Evidence that supports the same relation for futures is provided by Clark (1973), Tauchen and Pitts (1983), Garcia, Leuthold, and Zapata (1986), Ragunathan and Peker (1997), and Gwilym, MacMillan, and Speight (1999). Collectively, these studies clearly support the theoretical prediction of a positive and contemporaneous relationship between trading volume and volatility. This result is a common empirical finding for most financial assets, as Karpoff (1987) showed when he summarized the results of several studies on the positive relation between price changes and trading volume for commodity futures, currency futures, common stocks, and stock indices.

Foster (1995) concluded that not only is trading volume important in determining the rate of information (i.e. any news that affects the market), but also lagged volume plays a role. Although contemporary trading volume is positively related to volatility, lagged trading volume presents a negative relationship. Empirically, investigating daily data for several indices such as the S&P 500 futures contract, Wang and Yau (2000) observe that there is indeed a negative link between lagged trading volume and intraday price volatility. This means that an increase in trading volume today (as a measure of liquidity) will imply a reduction in price volatility tomorrow. In their study of five currency futures contracts, Fung and Patterson (1999) do in fact find a negative relationship between return volatility and past trading volume. In their view, the reversal behaviour of volatility with trading volume is generally consistent with the overreaction hypothesis (see Conrad *et al*. (1994)), and supports the sequential information hypothesis (see Copeland (1976)), which explains the relationship between return volatility and trading volume.

Despite the considerable amount of research in this area, there are no studies that use trading volume in an effort to improve the capability of models to forecast one-day-ahead VaR. Typically, in a VaR context, trading volume is only employed as a proxy for "liquidity risk" — the risk associated with trying to close out a position. In this paper, in contrast to prior studies, we analyze the impact of introducing trading volume on the ability to enhance performance in forecasting VaR one day ahead. We empirically test whether the introduction of trading volume will reduce the number of violations (i.e., the number of times when the estimated loss exceeds the observed one) in the spot and futures equity markets of the U.S., Germany, and Japan.

The remainder of this paper is organized as follows. ARMA-GARCH models with normal and tempered stable innovations are reviewed in Section 2. In Section 3, we discuss parameter estimation of the ARMA-GARCH models and forecasting daily return distributions. VaR values and backtesting of the ARMA-GARCH models are also reported in Section 2, along with a comparison of the results for (1) the spot and futures markets and (2) the normal and tempered stable innovations. Trading volume is introduced into the ARMA-GARCH model with tempered stable innovations in Section 4. VaR and backtesting of the ARMA-GARCH with different variants of trading volume are presented and compared to the results for models with and without trading volume. We summarize our principal findings in Section 5.

2. ARMA-GARCH model with normal and tempered stable innovations

In this section, we provide a review of the ARMA-GARCH models with normal and tempered stable innovations. For a more detailed discussion, see Kim et al. (2010).

Let $(S_t)_{t\geq 0}$ be the asset price process and $(y_t)_{t\geq 0}$ be the return process of $(S_t)_{t\geq 0}$ defined by $y_t = \log \frac{S_t}{S_{t-1}}$. The ARMA(1,1)-GARCH(1,1) model is:

$$\begin{cases} y_t = a y_{t-1} + b\sigma_{t-1}\varepsilon_{t-1} + \sigma_t\varepsilon_t + c_t \\ \sigma_t = \alpha_0 + \alpha_1 {\sigma^2}_{t-1}{\varepsilon^2}_{t-1} + \beta_1 {\sigma^2}_{t-1} \end{cases} \tag{1}$$

where $\varepsilon_0 = 0$, and a sequence $(\varepsilon_t)_{tcN} = 0$ of independent and identically distributed (iid) real random variables. The innovation ε_t is assumed to follow the standard normal distribution. This ARMA(1,1)-GARCH(1,1) model is referred to as the "normal-ARMA-GARCH model."

If the ε_t's are assumed to be tempered stable innovations, then we obtain a new ARMA(1,1)-GARCH(1,1) model. In this paper, we will consider the standard classical tempered stable (denoted by stdCTS) distributions. This ARMA(1,1)-GARCH(1,1) model is defined as follows: CTS-ARMA-GARCH model: $\varepsilon_t \sim stdCTS(\alpha, \lambda_+, \lambda_-)$. This distribution does not have a closed-form solution for its probability density function. Instead, it is defined by its characteristic function as follows: Let $\alpha \epsilon\ (0,2)\ \{1\}, C, \lambda_+, \lambda_- > 0$, and $m \in \mathbb{R}$. Then a random variable X is said to follow the classical tempered stable (CTS) distribution if the characteristic function of X is given by

$$\begin{aligned} \phi_X(u) &= \phi_{CTS}(u: \alpha, C, \lambda_+, \lambda_-, m) \\ &= exp\Big(ium - iuCT(1-\alpha)(\lambda_+^{\alpha-1} - \lambda_-^{\alpha-1})\Big) \\ &+CT(-\alpha)((\lambda_+ - iu)^\alpha - \lambda_+^\alpha + (\lambda_- - iu)^\alpha - \lambda_-^\alpha), \end{aligned} \tag{2}$$

and we denote $X{\sim}CTS(\alpha, C, \lambda_+, \lambda_-, m)$.

The cumulants of X are defined by

$$c_n(X) = \frac{\partial^n}{\partial u^n} \log E\big[\in^{iuX}\big]|_{u=0} \text{ , n=1,2,3,... .}$$

For the tempered stable distribution, we have $E[X] = c_1(X) = m$. The cumulants of the tempered stable distribution for *n* =2, 3, … are

$$c_n(X) = CT(n-\alpha)(\lambda_+^{\alpha-n} + (-1)^n \lambda_-^{\alpha-n}).$$

By substituting the appropriate value for the two parameters *m* and *C* into the three tempered stable distributions, we can obtain tempered stable distributions with zero mean and unit variance. That is,

$X{\sim}CTS(\alpha, C, \lambda_+, \lambda_-, 0)$ has zero mean and unit variance by substituting

$$C = \Big(\Gamma(2-\alpha)(\lambda_+^{\alpha-2} + \lambda_-^{\alpha-2})\Big)^{-1}. \tag{3}$$

The random variable *X* is referred to as the standard CTS distribution with parameters $(\alpha, \lambda_+, \lambda_-)$ and denoted by $X{\sim}stdCTS(\alpha, \lambda_+, \lambda_-)$.

3. VaR for the ARMA-GARCH model

In this section, we discuss VaR for the ARMA-GARCH model with normal and tempered stable innovations.

3.1 VaR and backtesting

The definition of VaR for a significance level η is

$$VaR_\eta(X) = -inf\{x \in \mathbb{R} | P(X \le x) > \eta\}.$$

If we take the ARMA-GARCH model described in Section 2, we can define VaR for the information until time *t* with significance level η as

$$VaR_{t,\eta}(y_{t+1}) = -inf\{x \in \mathbb{R} | P_t(y_{t+1} \le x) > \eta\}.$$

where $P_t(A)$ is the conditional probability of a given event A for the information until time *t*.

Two models are considered: normal-ARMA(1,1)-GARCH(1,1) and stdCTS-ARMA(1,1)-GARCH(1,1). For both models, the parameters have been estimated for the time series between December 14, 2004 and December 31, 2008. For each daily estimation, we worked with 10 years of historical daily performance for the S&P 500, DAX 30, and Nikkei 225 spot and futures indices. More specifically, we used daily returns calculated based on the closing price of those indices. In the case of futures indices, we constructed a unique continuous-time series using the different maturities of each futures index following the methodology proposed by Carchano and Pardo (2009).[1] Then, we computed VaRs for both models.

The maximum likelihood estimation method (MLE) is employed to estimate parameters of the normal-ARMA(1,1)-GARCH(1,1) model. For the CTS distribution, the parameters are estimated as follows:

1. Estimate parameters $\alpha_0, \alpha_1, \beta_1, a, b, c$ with normal innovations by the MLE.
2. Extract residuals using those parameters.
3. Fit the parameters of the innovation distribution (CTS) to the extracted residuals using MLE.

In order to determine the accuracy of VaR for the two models, backtesting using *Kupiec's proportion of failures test* (Kupiec, 1995) is applied. We first calculate the number of violations. Then, we compare the number of violations with the conventional number of exceedances at a given significance level. In Table 1 the number of violations and *p*-values for Kupiec's backtest for the three stock indices over the four one-year periods are reported. Finally, we sum up the number of violations and their related *p*-values for 1%-VaRs for the normal and CTS ARMA-GARCH models.

Based on Table 1, we conclude the following for the three stock indices. First, a comparison of the normal and tempered stable models indicates that there are no cases using the tempered stable model at the 5% significance level, whereas the normal model is rejected five times. This evidence is consistent with the findings of Kim et al. (2010). Second, a comparison of the spot and futures indices indicates that spot data provide less than or the same number of violations than futures data. One potential explanation is that futures markets are more volatile, particularly, when the market falls.[2] This overreaction to bad news could cause the larger number of violations.

4. Introduction of trading volume

In the previous section, we showed the usefulness of the tempered stable model for stock index futures. Motivated by the vast literature linking trading volume and volatility, for the first time we investigate whether the introduction of trading volume in the CTS model could improve its ability to forecast one-day-ahead VaR.

Let $(S_t)_{t\geq 0}$ be the asset price process and $(y_t)_{t\geq 0}$ be the return process of $(S_t)_{t\geq 0}$ defined by $y_t = \log \frac{S_t}{S_{t-1}}$. We propose the following ARMA(1,1)-GARCH(1,1) with trading volume model:

$$\begin{cases} y_t = a y_{t-1} + b\sigma_{t-1}\varepsilon_{t-1} + \sigma_t\varepsilon_t + c \\ \sigma_t = \alpha_0 + \alpha_1 {\sigma^2}_{t-1}{\varepsilon^2}_{t-1} + \beta_1 {\sigma^2}_{t-1} + \gamma_1 Vol_{t-1}, \end{cases} \quad (4)$$

[1] Thus, the last trading day of the front contract is chosen as the rollover date. Then, the return of the day after the rollover date is calculated as the quotient between the closing price of the following contract and the previous closing price of such contract. By doing so, all the returns are taken from the same maturity.

[2] We compared the spot and futures series when the markets discount bad news (negative returns). We find that for the three stock indices, futures volatility is significantly greater than spot volatility at a 5% significance level. Moreover, for all three stock indices, the minimum return and the 1%-percentile return are also lower for futures data than spot data.

where $\varepsilon_0 = 0$, and a sequence $(\varepsilon_t)_{tcN} = 0$ of *iid* real random variables. The innovation ε_t is assumed to be the tempered stable innovation. We will consider the standard classical tempered stable distributions. This new ARMA(1,1)-GARCH(1,1)-V model is defined as follows:

$$\text{CTS-ARMA-GARCH-V model: } \varepsilon_t \sim stdCTS(\alpha, \lambda_+, \lambda_-).$$

4.1. Different variants of trading volume

For the S&P 500 cash and futures markets, we test the following versions of trading volume in order to determine which one would be the most appropriate:

- Lagged trading volume in levels: V(*t*-1)
- Logarithm of lagged trading volume: Log[V(*t*-1)]
- Relative change of lagged trading volume: Ln[V(*t*-1)/V(t-2)]

The spot series trading volume is in dollars; for the futures series, the trading value is in number of contracts. We can calculate the volume of the futures market in dollars too. The tick value of the S&P 500 futures contract is 0.1 index points or $25. Multiplying the number of contracts by the price, and finally by $250 (the contract's multiple), we obtain the trading volume series for the futures contract in dollars. Thus, for the futures contract we get three new versions of trading volume to test:

- Lagged trading volume in dollars: V$(*t*-1)
- Logarithm of trading volume in dollars: Log[V$(*t*-1)]
- Relative change of lagged trading volume in dollars: Ln[V$(*t*-1)/V$(*t*-2)]

By doing that, we can determine which series (in dollars or in contracts) seems to be more useful for the futures index.

In Table 2 we report the number of violations and *p*-values of Kupiec's backtest for the different versions of the CTS-ARMA-GARCH-V model for the S&P 500 spot and futures indices. We count the number of violations and the corresponding *p*-values for 1%-VaRs of both markets. From Table 2, we conclude the following:

- The model with the lagged trading volume in level is rejected at the 1% significance level in all four years for the S&P 500 spot, and for the second period (2005-2006) for the S&P 500 futures.

- The logarithm of trading volume in the model is rejected at the 5% significance level for the spot market for the third period (2006-2007), but it is not rejected in any period for the futures market.

- The relative change of the lagged volume is not rejected at the 5% significance level in any period in either market. Of the three versions of trading volume tests, this version seems to be the most useful for both spot and futures markets.

- The results for trading volume in contracts and the trading volume in dollars in the futures market indicate that the former is rejected at the 1% significance level only for the lagged trading volume in level in the second period(2005- 2006). Trading volume in dollars is rejected three times, for the lagged trading volume in levels for the third period (2006-2007), and for the lagged relative trading volume change in the last two periods (2006-2007, and 2007-2008). These findings suggest that the trading volume in contracts is the preferred measure.

4.2. Lagged relative change of trading volume.

As we have just seen, the variant of trading volume that seems more useful for forecasting one-day-ahead VaR using CTS-ARMA-GARCH is the relative change of trading volume. Next, we compare the original CTS-ARMA-GARCH model with the new CTS-ARMA-GARCH-V model where V is the lagged relative change of trading volume. Table 3 shows the number of violations and *p*-values of Kupiec's backtest for the two models for the three stock indices and both markets. We sum up the number of violations and the corresponding *p*-values for 1%-VaRs for each case.

Our conclusions from Table 3 are as follows. For the spot markets, the introduction of trading volume does not mean a reduction in the number of violations in any period for any index. However, for the futures markets, the numbers of violations are the same or lower for the model with trading volume than with the original model. Thus, by introducing trading volume, we get a slightly more conservative model, increasing the VaR forecasted for futures equity markets.

4.3. Lagged trading volume or forecasting contemporaneous trading volume.

Although there is some evidence which supports the relationship between lagged trading volume and volatility, the literature is not as extensive as the studies that establish a strong link between volatility and contemporaneous trading volume. As there are countless ways to try to forecast trading volume, we begin by introducing contemporaneous trading volume relative change in the model as a benchmark to assess whether it is worthwhile to forecast trading volume.

In Table 4 we show the number of violations and *p*-values of Kupiec's backtest for the CTS-ARMA-GARCH with contemporaneous and lagged relative change of trading volume for the three stock indices for both markets. We count the number of violations and the corresponding p-values for 1%-VaRs for the six indices.

Our conclusions based on the results reported in Table 4 are as follows. First, with the exception of the S&P 500 futures, the introduction of the contemporaneous relative change of trading volume in the model is rejected at the 1% significance level for the last period analyzed (2007-2008). In the case of the S&P 500 futures, it is rejected at the significance level of 5% for the third period (2006-2007). Second, the model with lagged relative change of trading volume is not rejected for any stock index or market. It seems to be more robust than contemporaneous trading volume (although, in general, there are fewer violations when using it).

Our results suggest that it is not worth making an effort to predict contemporaneous trading volume because the forecasts will be flawed and two variables would have to be predicted (VaR and contemporaneous trading volume). Equivalently, the lagged trading volume relative change appears to be more robust because it is not rejected in any case, although it provides a poor improvement to the model.

5. Conclusions

Based on an empirical analysis of spot and futures trading for the S&P 500, DAX 30, and Nikkei 225 stock indices, in this paper we provide empirical evidence about the usefulness of using classical tempered stable distributions for predicting one-day-ahead VaR. Unlike prior studies that investigated CTS models in the cash equity markets, we analyzed their suitability for both spot markets and futures markets. We find in both markets the CTS models perform better in forecasting day-ahead VaR than models that assume innovations follow the normal law.

Second, we introduced trading volume into the CTS model. Our empirical evidence suggests that lagged trading volume relative change provides a slightly more conservative model (i.e., reduces the number of violations) to predict one-day-ahead VaR for stock index futures contracts. We cannot state the same for the cash market because the results are mixed depending on the index. After that, we introduced contemporaneous trading volume in order to improve the forecasting ability of the model. We find that does not seem to be worth the effort. That is, trading volume appears not to offer enough information to improve forecasts.

Finally, we compared the number of violations of the estimated VaR in the spot and futures equity markets. For the CTS model without volume, in general, we find fewer violations in the spot indices than in the equivalent futures contracts. In contrast, our results suggest that the number of violations in futures markets is less in the case of the CTS model with trading volume in comparison to the CTS model that ignores trading volume. But if we contrast spot and futures equity markets, violations are still greater for futures than in spot markets. A possible reason is that futures markets demonstrate extra volatility or an overreaction when the market falls with respect to their corresponding spot markets.

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Table 1: Number of violations (*N*) and p values of Kupiec's proportion of failures test for the S&P 500, DAX 30 and Nikkei 225 spot and futures indices data.

Model	1 year (255 days)			
	Dec. 14, 2004 ~ Dec. 15, 2005	Dec. 16, 2005 ~ Dec. 20, 2006	Dec. 21, 2006 ~ Dec. 27, 2007	Dec. 28, 2007 ~ Dec. 31, 2008
	N(*p*-value)	*N*(*p*-value)	*N*(p-value)	*N*(*p*-value)
		S&P 500 Spot		
Normal-ARMA-GARCH	1(0.2660)	3(0.7829)	8(0.0061)	10(0.0004)
CTS-ARMA-GARCH	0	2(0.7190)	6(0.0646)	4(0.3995)
		S&P 500 Futures		
Normal-ARMA-GARCH	3(0.7829)	3(0.7829)	7(0.0211)	9(0.0016)
CTS-ARMA-GARCH	1(0.2660)	3(0.7829)	4(0.3995)	5(0.1729)
		DAX 30 Spot		
Normal-ARMA-GARCH	4(0.3995)	4(0.3995)	3(0.7829)	6(0.0646)
CTS-ARMA-GARCH	4(0.3995)	4(0.3995)	3(0.7829)	4(0.3995)
		DAX 30 Futures		
Normal-ARMA-GARCH	3(0.7829)	5(0.1729)	6(0.0646)	6(0.0646)
CTS-ARMA-GARCH	3(0.7829)	4(0.3995)	6(0.0646)	3(0.7829)
		Nikkei 225 Spot		
Normal-ARMA-GARCH	2(0.7190)	4(0.3995)	5(0.1729)	5(0.1729)
CTS-ARMA-GARCH	1(0.2660)	3(0.7829)	4(0.3995)	5(0.1729)
		Nikkei 225 Futures		
Normal-ARMA-GARCH	2(0.7190)	2(0.7190)	7(0.0211)	5(0.1729)
CTS-ARMA-GARCH	5(0.1729)	5(0.1729)	6(0.0646)	6(0.0646)

Table 2: Number of violations (*N*) and *p* values of Kupiec's proportion of failures test for the S&P 500 spot and futures indices with the different variants of volume into the stdCTS-ARMA(1,1)-GARCH(1,1) model.

Model	1 year (255 days)			
	Dec. 14, 2004 ~ Dec. 15, 2005	Dec. 16, 2005 ~ Dec. 20, 2006	Dec. 21, 2006 ~ Dec. 27, 2007	Dec. 28, 2007 ~ Dec. 31, 2008
	N(*p*-value)	*N*(*p*-value)	*N*(p-value)	*N*(*p*-value)
	S&P 500 Spot			
CTS-ARMA-GARCH-V(*t*-1)	16(0.0000)	10(0.0004)	23(0.0000)	26(0.0000)
CTS-ARMA-GARCH-log[V(*t*-1)]	1(0.2660)	4(0.3995)	10(0.0004)	6(0.0646)
CTS-ARMA-GARCH-ln[V(*t*-1)/V(*t*-2)]	0	2(0.7190)	6(0.0646)	5(0.1729)
	S&P 500 Futures			
CTS-ARMA-GARCH-V(*t*-1)	1(0.2660)	11(0.0001)	4(0.3995)	6(0.0646)
CTS-ARMA-GARCH-log[V(*t*-1)]	1(0.2660)	3(0.7829)	4(0.3995)	5(0.1729)
CTS-ARMA-GARCH-ln[V(*t*-1)/V(*t*-2)]	1(0.2660)	3(0.7829)	3(0.7829)	5(0.1729)
CTS-ARMA-GARCH-V$(*t*-1)	0	1(0.2660)	15(0.0000)	3(0.7829)
CTS-ARMA-GARCH-log[V$(*t*-1)]	3(0.7829)	3(0.7829)	4(0.3995)	5(0.1729)
CTS-ARMA-GARCH-ln[V$(*t*-1)/V$(*t*-2)]	2(0.7190)	0	8(0.0061)	8(0.0061)

Notes:
V(*t*-1), log[V(*t*-1)] and ln[V(*t*-1)/V(*t*-2)] stand for levels, logarithm and relative change of the lagged trading volume, respectively.
V$(*t*-1), log[V$(*t*-1)] and ln[V$(*t*-1)/V$(*t*-2)] stand for levels, logarithm and relative change of the lagged trading volume in dollars, respectively.

Table 3: Number of violations (*N*) and *p* values of Kupiec's proportion of failures test for the S&P 500, DAX 30 and Nikkei 225 spot and futures indices. CTS-ARMA-GARCH and CTS-ARMA-GARCH with lagged relative change of trading volume are compared.

Model	1 year (255 days)			
	Dec. 14, 2004 ~ Dec. 15, 2005	Dec. 16, 2005 ~ Dec. 20, 2006	Dec. 21, 2006 ~ Dec. 27, 2007	Dec. 28, 2007 ~ Dec. 31, 2008
	N(*p*-value)	*N*(*p*-value)	*N*(*p*-value)	*N*(*p*-value)
	S&P 500 Spot			
CTS-ARMA-GARCH	0	2(0.7190)	6(0.0646)	4(0.3995)
CTS-ARMA-GARCH-ln[V(*t*-1)/V(*t*-2)]	0	2(0.7190)	6(0.0646)	5(0.1729)
	S&P 500 Futures			
CTS-ARMA-GARCH	1(0.2660)	3(0.7829)	4(0.3995)	5(0.1729)
CTS-ARMA-GARCH-ln[V(*t*-1)/V(*t*-2)]	1(0.2660)	3(0.7829)	3(0.7829)	5(0.1729)
	DAX 30 Spot			
CTS-ARMA-GARCH	4(0.3995)	4(0.3995)	3(0.7829)	4(0.3995)
CTS-ARMA-GARCH-ln[V(*t*-1)/V(*t*-2)]	5(0.1729)	4(0.3995)	3(0.7829)	5(0.1729)
	DAX 30 Futures			
CTS-ARMA-GARCH	3(0.7829)	4(0.3995)	6(0.0646)	3(0.7829)
CTS-ARMA-GARCH-ln[V(*t*-1)/V(*t*-2)]	3(0.7829)	4(0.3995)	5(0.1729)	3(0.7829)
	Nikkei 225 Spot			
CTS-ARMA-GARCH	1(0.2660)	3(0.7829)	4(0.3995)	5(0.1729)
CTS-ARMA-GARCH-ln[V(*t*-1)/V(t-2)]	3(0.7829)	3(0.7829)	4(0.3995)	5(0.1729)
	Nikkei 225 Futures			
CTS-ARMA-GARCH	5(0.1729)	5(0.1729)	6(0.0646)	6(0.0646)
CTS-ARMA-GARCH-ln[V(*t*-1)/V(*t*-2)]	4(0.3995)	4(0.3995)	6(0.0646)	6(0.0646)

Table 4: Number of violations (*N*) and *p*-values of Kupiec's proportion of failures test for the S&P 500, DAX 30, and Nikkei 225 spot and futures indices. CTS-ARMA-GARCH with the relative change of trading volume and CTS-ARMA-GARCH with lagged relative change of trading volume are compared.

Model	1 year (255 days)			
	Dec. 14, 2004 ~ Dec. 15, 2005	Dec. 16, 2005 ~ Dec. 20, 2006	Dec. 21, 2006 ~ Dec. 27, 2007	Dec. 28, 2007 ~ Dec. 31, 2008
	N(*p*-value)	*N*(*p*-value)	*N*(*p*-value)	*N*(*p*-value)
	S&P 500 Spot			
CTS-ARMA-GARCH-ln[V(*t*)/V(*t*-1)]	0	3(0.7829)	3(0.7829)	8(0.0061)
CTS-ARMA-GARCH-ln[V(*t*-1)/V(*t*-2)]	0	5(0.1729)	6(0.0646)	5(0.1729)
	S&P 500 Futures			
CTS-ARMA-GARCH-ln[V(*t*)/V(*t*-1)]	0	1(0.2660)	7(0.0211)	6(0.0646)
CTS-ARMA-GARCH-ln[V(*t*-1)/V(*t*-2)]	1(0.2660)	3(0.7829)	3(0.7829)	5(0.1729)
	DAX 30 Spot			
CTS-ARMA-GARCH-ln[V(*t*)/V(*t*-1)]	0	1(0.2660)	3(0.7829)	11(0.0001)
CTS-ARMA-GARCH-ln[V(*t*-1)/V(*t*-2)]	5(0.1729)	4(0.3995)	3(0.7829)	5(0.1729)
	DAX 30 Futures			
CTS-ARMA-GARCH-ln[V(*t*)/V(*t*-1)]	1(0.2660)	1(0.2660)	2(0.7190)	8(0.0061)
CTS-ARMA-GARCH-ln[V(*t*-1)/V(*t*-2)]	3(0.7829)	4(0.3995)	5(0.1729)	3(0.7829)
	Nikkei 225 Spot			
CTS-ARMA-GARCH-ln[V(*t*)/V(*t*-1)]	3(0.7829)	5(0.1729)	7(0.0211)	8(0.0061)
CTS-ARMA-GARCH-ln[V(*t*-1)/V(*t*-2)]	3(0.7829)	3(0.7829)	4(0.3995)	5(0.1729)
	Nikkei 225 Futures			
CTS-ARMA-GARCH-ln[V(*t*)/V(*t*-1)]	1(0.2660)	1(0.2660)	3(0.7829)	11(0.0001)
CTS-ARMA-GARCH-ln[V(t-1)/V(*t*-2)]	4(0.3995)	4(0.3995)	6(0.0646)	6(0.0646)