PORTFOLIO SELECTION BASED ON A SIMULATED COPULA

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Abstract: In this paper, we propose a methodology to value the portfolio choices based on the prediction of future returns where the dependence structure of joint returns and the behavior of single returns are estimated separately. In particular, we assume the marginals evolve as an ARMA(0,2)-GARCH(0,2) model with stable paretian residuals and the joint distribution of residuals is estimated with an asymmetric t-copula. Then, we compare the ex-post final wealth sample paths of different strategies based on reward/risk ratios. Doing so we examine and discuss the impact of the forecasting method with respect to classic myopic portfolio strategies based on the same reward/risk ratios.
Key words: performance ratios, asymmetric t copula, stable distributions, dynamic mesures.

1 Introduction

This paper analyses and discusses the profitability of some reward risk strategies based on a forecasted evolution of returns. In particular, we examine the impact of a simulated copula on the investors choices. We first try to approximate as much as possible the joint behavior of future returns taking into account their distributional characteristics. Then we compare ex post the choices based on this approximations.

The motivations behind this paper comes from three stylized facts about real world financial markets. First, financial return series are asymmetric and heavy tailed and they cannot be approximated with a normal distribution that is symmetric and has too light tails to match market data. Second, there is volatility clustering in time series since calm periods are generally followed by highly volatile periods and vice versa. Finally, a dependence structure of multivariate distribution is needed beyond simple linear correlation. The dependence model has to be flexible enough to account for several empirical phenomena observed in the data, in particular asymmetry of dependence and dependence of the tail events (see among others Rachev and Mittnik (2000), Rachev et al. (2005), Rachev et al. (2007)). Therefore the dependence model cannot be approximated with a multivariate normal distribution that fails to describe both phenomena (i.e., the covariance is a symmetric dependence concept and the tail events are asymptotically independent). In searching for an acceptable model to describe these three stylized facts, we examine the behavior of marginals with a time series process belonging to the ARMA-GARCH family with stable paretian innovations and we suggest to model dependencies with an asymmetric t copula valued on the innovation of the marginals. As a matter of fact, the use of a copula function allows to take into account of phenomena such as clustering of the volatility effect, heavy-tails, and skewness and then separately model the dependence structure between them. By a practical point of view, the problem to fit the model to the market data can be solved in two steps, indeed marginals and copula can be estimated separately.

In order to value the impact of this methodology that simulate the joint behavior of future returns we compare the performance of several reward risk strategies based either on simulated data or on historical ones. In particular, we use the STARR ratio (see Martin et al. (2003)), and we introduce two new reward/risk ratios based on some dynamic measures recently proposed in literature (see, Rachev et al. (2008), Chekhlov, et al (2005)). As we expect the comparison confirms the better performance of strategies valued on simulated data with respect to those valued on historical data.

The remainder of the paper is organized as follows: Section 2 provides a brief description of the methodology to build scenarios based on a simulated copula. Section 3 provides a comparison among different strategies and Section 4 concludes the paper.

2 Generation of scenarios based on ARMA-GARCH and Copula Models

So far a systematic methodology to forecast, control and model portfolios in volatile markets has not been developed. If we observe the behaviour of the returns we notice several anomalies: heavy tailed distributions, volatility clustering, non Gaussian copula dependence (see, among others, Rachev and Mittnik (2000), Rachev et al. (2005), Rachev et al. (2007)). The empirical evidence remarks the opportunity and the necessity of properly considering the dependence structure of financial variables. In order to describe the dependence structure we discuss a copula approach combined with an opportune valuation of the single return series.

2.1 Methodology to model time series

Let us consider the problem of time series modeling. We propose the following model to simulate future scenarios taking into account the financial structure of the market.

- **Step 1.** Assume we have d series of size T of (closing) daily returns, i.e., $r_t^{(j)}$ is the daily return of the asset j (j=1,..., d) at day t (t=1,...,T).
- **Step 2.** Carry out maximum likelihood parameter estimation of ARMA(p,q)-GARCH(s,u) of each serie:

$$r_t^{(j)} = a_{j,0} + \sum_{i=1}^{p} a_{j,i} r_{t-i}^{(j)} + \sum_{i=1}^{q} b_{j,i}\varepsilon_{j,t-i} + \varepsilon_{j,t} \tag{1}$$

$$\varepsilon_{j,t} = \sigma_{j,t} z_{j,t} \tag{2}$$

$$\sigma_{j,t}^2 = K_j + \sum_{i=1}^{s} c_{j,i}\sigma_{j,t-i}^2 + \sum_{i=1}^{u} e_{j,i}\varepsilon_{j,t-i}^2; \tag{3}$$

$$j = 1, ..., d;\ t = 1, ..., T.$$

- **Step 3.**Approximate with α_j-stable distribution $S_{\alpha_j}(\sigma_j, \beta_j, \mu_j)$ the empirical standardized innovations

$$\widehat{z}_{j,t} = \widehat{\varepsilon}_{j,t}/\sigma_{j,t} \tag{4}$$

where the innovations

$$\widehat{\varepsilon}_{j,t} = r_t^{(j)} - a_{j,0} - \sum_{i=1}^{p} a_{j,i} r_{t-i}^{(j)} - \sum_{i=1}^{q} b_{j,i}\varepsilon_{j,t-i};\ j = 1, ..., d$$

are obtained from (1) (we refer to Samorodnitsky and Taqqu (1994) and Rachev and Mittnik (2000), for a general discussion on properies and use of stable distributions).

- **Step 4.**Simulate S stable distributed scenarios for each of the future standardized innovations series and compute the sample distribution functions of these simulated series:

$$F_{\widehat{z}}^{(j)}(x) = \frac{1}{S}\sum_{s=1}^{S} I_{\left\{\widehat{u}_{T+1}^{(j,s)} \leq x\right\}},\ x \in \Re,\ j = 1, ..., d$$

where $\widehat{u}_{T+1}^{(j,s)}(\ 1 \leq s \leq S)$ is the s-th value simulated with the fitted α_j-stable distribution for future standardized innovation (valued in $T+1$) of the *j-th* asset.

Remark: Steps 3-4 provide the marginal distributions for standardized innovations each of the d assets used to simulate the next-period returns. In the following steps we will estimate the dependence structure of the vector of standardized innovations with an asymmetric t-copula.

- **Step 5** Fit the d-dimensional vector of empirical standardized innovations

$$\widehat{z} = [\widehat{z}_1, ..., \widehat{z}_d]'$$

obtained by (4) with a d-dimensional asymmetric t-distribution[1]. So, let the d-dimensional vector $V = [V_1, ..., V_d]'$ be asymmetric t-distributed. It has the form:

$$V = \mu Y + \sigma Y \cdot Z \tag{5}$$

where μ is a constant, σ is a positive constant, $Y \cdot Z = [Y^{(1)}Z^{(1)}, ..., Y^{(d)}Z^{(d)}]'$, and $Y = [Y^{(1)}, ..., Y^{(d)}]'$ is a d-dimensional vector with positive components distributed as: $Y^{(j)} \overset{d}{=} \frac{1}{\sqrt{\chi^2(\upsilon_j)}}$, where $\chi^2(\upsilon_j)$ is chi-squared distributed random variable with υ_j degrees of freedom. We assume that the components $Y^{(j)}$ $(j = 1, ..., d)$ are independent and vector Y is independent of the vector $Z = [Z^{(1)}, ..., Z^{(d)}]'$. The vector Z is normally distributed with zero mean and covariance matrix Σ. We use the maximum likelihood method to estimate all the parameters of the asymmetric t-distribution $F_V(x_1, ..., x_d)$ for $\widehat{z}$, given by (4). Thus, the asymmetric t-copula is given by:

$$C(t_1, ..., t_d) = F_V(F_{V_1}^{-1}(t_1), ..., F_{V_d}^{-1}(t_d)); 0 \leq t_i \leq 1; 1 \leq i \leq d \tag{6}$$

where $F_{V_i}^{-1}(t_i)$ is the left inverse of the *i-th* marginal distribution of F_V.

- **Step 6.** Since we have estimated all the parameters of Y and Z as well as the constants μ and σ we can generate S scenarios for Y and, independently, S scenarios for Z, and using (5) we obtain S scenarios for the vector of standardized innovations z, that is asymmetric t-distributed. Denote these scenarios $(V_1^{(s)}, ..., V_d^{(s)})$, $s = 1, ..., S$ and denote the marginal

[1] Clearly many other possible heavy tailed, asymmetric multivariate distributions can be considered in this step.

distributions $F_{V_j}(x)$, $1 \leq j \leq d$ of the estimated d-dimensional asymmetric t- distribution:

$$F_V(x_1, ..., x_d) = P(V_1 \leq x_1, ..., V_d \leq x_d).$$

Then considering $U_j^{(s)} = F_{V_j}(V_j^{(s)})$, $1 \leq j \leq d, 1 \leq s \leq S$, we can generate S scenarios $(U_1^{(s)}, ..., U_d^{(s)})$, $s = 1, ..., S$ of the uniform random vector $(U_1, ..., U_d)$ that has support on the d-dimensional unit cube and whose distribution is given by the copula $C(t_1, ..., t_d)$ of formula (6).

Remark: Steps 5-6 serve to estimate the dependence structure among the innovations with an asymmetric t-copula. The next step combines the marginal distributions and the scenarios for the copula into scenarios for the vector of returns.

- **Step 7.** On one side, we have determined the stable distributed marginal sample distribution function of the j-th standardized innovation $F_z^{(j)}(x)$, $j = 1, ..., d$ (see step 4), on the other side we have the scenarios $U_j^{(s)}$ for $1 \leq j \leq d; 1 \leq s \leq S$ (see step 6). Then we generate S scenario of the vector of standardized innovations $z_{T+1}^{(s)} = (z_{T+1}^{(1,s)}, ..., z_{T+1}^{(d,s)}), s = 1, ..., S$ valued at time $T + 1$ (taking into account of the dependence structure of the vector) assuming

$$z_{T+1}^{(j,s)} = \left(F_z^{(j)}\right)^{-1} \left(U_j^{(s)}\right); 1 \leq j \leq d; 1 \leq s \leq S,$$

  Doing so, we generate the vector of standardized innovation assuming that the marginal distributions are α_j-stable distributions and considering the copula dependence, given by (6).

- **Step 8** Once we have described the multivariate behavior of standardized innovation at time $T+1$ using relation (2) we can generate S scenario of the vector of innovation $\varepsilon_{T+1}^{(s)} = \left(\varepsilon_{T+1}^{(1,s)}, ..., \varepsilon_{T+1}^{(d,s)}\right) = \left(\sigma_{1,T+1} z_{T+1}^{(1,s)}, ..., \sigma_{d,T+1} z_{T+1}^{(d,s)}\right)$, $s = 1, ..., S$ where $\sigma_{j,T+1}$ are defined by (3). Thus, using relation (1), we can generate S scenario of the vector of returns

$$r_{T+1,s} = [r_{T+1}^{(1,s)}, ..., r_{T+1}^{(d,s)}]'.$$

Observe steps 4-7 can be always used to generate a distribution with some given marginals and a given dependence structure (see among others Rachev et al. (2005), Sun et al. (2008a-2008b), Biglova et al. (2008), Cherubini et al. (2004) for the definition of some classical copula used in finance literature)..

3 An empirical comparison among some reward/risk ratios

Let us consider the optimal portfolio choice problem among d risky assets assets with log-returns $r_t = [r_t^{(j)}, ..., r_t^{(d)}]'$ and assume that there exist a risk-free benchmark with log-return $r_{t,b}$ all valued at time t. When no short selling is allowed, every portfolio of returns is a convex combination of the risky log-returns $r_t^{(j)}$ i.e., $x'r_t = \sum_{i=1}^{d} x_i r_t^{(i)}$ where $x = (x_1, ..., x_d)' \in C = \left\{ y \in \Re^d \,\middle|\, y_i \geq 0, \sum_{i=1}^{d} y_i = 1 \right\}$ is the vector of weights. In order to value the impact of the previous model, we provide an empirical ex-post comparison among several strategies based on simulated data and on historical one. As initial data we use the daily return series of the benchmark three months treasury bill and n =5 daily returns on US stock indexes [2] from 12/14/1992 till 2/27/2008 for a total of 3831 observations.

First of all we decide the type of ARMA-GARCH model should be used for the simulation of future scenarios. From this preliminary analysys we deduce that the above asset returns are well approximated by an ARMA(2,0)-GARCH(0,2) model. That is, for each series ($j = 1, ..., 5$) the formulas (1, 2, 3) are represented by:

$$r_t^{(j)} = C + AR(1) r_{t-1}^{(j)} + AR(2) r_{t-2}^{(j)} + \varepsilon_t$$

$$\varepsilon_{j,t} = \sigma_{j,t} z_{j,t}$$
$$\sigma_{j,t}^2 = K + ARCH(1)\varepsilon_{j,t-1}^2 + ARCH(2)\varepsilon_{j,t-2}^2.$$

Then we suppose that decision makers invest their wealth purchasing the *market portfolio* taking into account that each investor has a diverse reward/risk perception. In the last years, several performance measures have been proposed and used in portfolio theory to capture the different perception of reward and risk (see Biglova et al. (2004) (2008)). Among these we recall the STARR ratio and we propose two other alternative performance measures. Thus, we assume that the market portfolio is determined by maximizing one of the following performance measures.

3.1 Performance measures

STARR ratio ($STARR$). The STARR ratio (see Martin et al. (2003)) serves to value the expected excess return for unity of risk represented by the Expected Tail Loss (ETL).i.e.:

$$STARR(x'r_{T+1}, \alpha) = \frac{E(x'r_{T+1} - r_{T+1,b})}{ETL_\alpha(x'r_{T+1} - r_{T+1,b})}.$$

The Expected Tail Loss , also known as Conditional Value-at-Risk (CVaR) , is defined as

$$ETL_\alpha(X) = \frac{1}{\alpha}\int_0^\alpha VaR_q(X)dq,$$

where $VaR_q(X) = -F_X^{-1}(q) = -\inf\{x \,|P(X \leq x) > q\}$ is the Value-at-Risk (VaR) of the random return X. If we assume a continuous distribution for the

[2] The indexes are: Dow Jones Industrials, NYSE, Major Market Index, DJ composite, DJAIG Commodity.

probability law of X, then $ETL_\alpha(X) = -E(X\,|X \leq -VaR_\alpha(X))$ and thus, ETL can be interpreted as the average loss beyond VaR. In our comparison we consider the $STARR$ ratio with parameters $\alpha = 0.01,\ 0.05$.

Rachev maximum drawup/down ratio (*R-maxdud*) With this dynamic performance ratio we introduce the concept of drawup measure and we suggest the use of the drawdown in a reward/risk plane. The absolute drawdown process $dd(x) = \{dd_t(x)\}_{t=1,...,T}$ of a portfolio x has been proposed in portfolio literature by Grossman and Zohu (1993), Cvitanic and Karatzas (1995) (see also Chechkov et al. (2005)) as function of the uncompunded cumulative excess portfolio rate $w_t(x)$ valued at time t, that is:

$$w_t(x) = \sum_{s=1}^{t} x' r_s - r_{t,b},\ \ t = 1, ..., T.$$

Analogously we can define the absolute drawup process $du(x) = \{du_t(x)\}_{t=1,...,T}$. Thus we call drawup and drawdown of the portfolio x respectively the processes given by:

$$du_t(x) = w_t(x) - \min_{s=1,...,t} w_s(x)$$

$$dd_t(x) = \max_{s=1,...,t} w_s(x) - w_t(x)$$

for $t = 1, ..., T$. Then the Rachev maximum drawup/down ratio is a performance ratio between the maximum drawup and the maximum drawdown that is

$$R - maxdud(x' r_{T+1}) = \frac{\max_{t=1,...,T} du_t(x)}{\max_{t=1,...,T} dd_t(x)}$$

Rachev average drawup/down ratio (*R-avedud*) The Rachev average drawup/down ratio is still defined as function of the absolute drawdown and drawup processes $dd(x)$, $du(x)$. However, instead of the maximum we consider the empirical mean, that is:

$$R - avedud(x' r_{T+1}) = \frac{\frac{1}{T}\sum_{t=1}^{T} du_t(x)}{\frac{1}{T}\sum_{t=1}^{T} dd_t(x)}$$

Moreover as in the Chechkov et al. (2005) analysis we can introduce futher ratios that value the ETL ratio of absolute drawdown and drawup processes, but these further performance measures will be object of future empirical analysis. Next we summarize the empirical comparison among different portfolio strategies.

3.2 A first ex-post empirical comparison among different portfolio strategies

Let us summarize the empirical ex-post comparison between historical and simulated data and among different portfolio strategies. For any optimal portfolio chosen on a daily basis, when we use the dynamic performance measures R-avedud and R-maxdud we consider a window of two years $T = 500$ of historical obervations, while we adopt a window of one year $T = 250$ when we use the $STARR$ ratio with parameters $\alpha = 0.01$, 0.05. We use these observations:

a) to compute the performance measures if we use historical data, and

b) to generate $S = T$ future scenarios of returns according to the algorithm proposed in the previous section when we use simulated data.

Therefore, for any reward/risk criterion measure $\rho(x'r)$, we can compute the optimal portfolio as solution the following optimization problem:

$$\begin{array}{c} \max\limits_{x \in C} \rho(x'r) \\ \text{subject to} \\ \sum_{i=1}^{n} x_i = 1; x_i \geq 0; i = 1, ..., n \end{array} \tag{7}$$

Since we assume that decision makers invest their wealth in the market portfolio (solution of (7)), we consider the sample path of the final wealth and of the cumulative return obtained from the different approaches. Then, we compare the efficiency of alternative performance measures. We assume that the investor has an initial wealth W_0 equal to 1 and an initial cumulative return CR_0 equal to 0 (at the date 12/5/1994 when we use $T = 500$ and at the date 12/8/1993 when we use $T = 250$) and at the k-th recalibration ($k = 0, 1, 2, ...$), three main steps are performed to compute the ex-post final wealth and cumulative return:

Step 1 Choose a performance ratio. Generate scenarios with the algorithm of the previous section if we use simulated data. Determine the market portfolio $x_M^{(k)}$ that maximizes the performance ratio $\rho(.)$ associated to the strategy, i.e. the solution of optimization problem of (7).

Step 2 The ex-post final wealth is given by:

$$W_{k+1} = W_k \left(\left(x_M^{(k)} \right)' (1 + r_{kT+1}) \right), \tag{8}$$

where r_{kT+1} is the vector of observed returns between kT and $kT{+}1$. The ex-post cumulative return is given by:

$$CR_{k+1} = CR_k + \left(x_M^{(k)} \right)' r_{kT+1}.$$

Step 3 The optimal portfolio $x_M^{(k)}$ is the new starting point for the $(k + 1)$-th optimization problem (7).

Steps 1, 2 and 3 are repeated until there are observations available and for all the performance ratios.

The output of this analysis is represented in Figures 1,2,3,4,5,6. All figures show a greater performance for the reward/risk ratios based on simulated data. STARR ratio with $\alpha = 0.05$ presents better performance with respect to STARR ratio with $\alpha = 0.01$ (see Figures 1,2). Moreover the most relevant impact is due

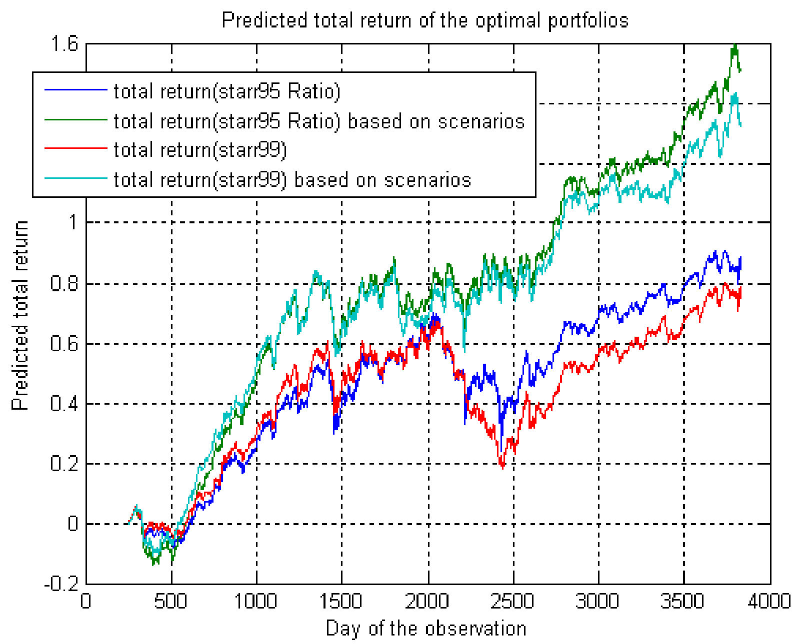

Figure 1: This figure compares the ex-post cumulative return obtained maximizing the STARR ratio (with $1-\alpha = 99\%, 95\%$) and using either real data or simulated data.

to the use of simulated data with respect to the historical observations. Figures 3,4 reports the ex-post final wealth and cumulative return processes when we use the Rachev average drawup/down ratio and even in this case we observe an higher final wealth and cumulative return using simulated data. This difference is a little bit greater (see Figures 5,6) when we use the Rachev maximum drawup/down ratio that presents better performance even with respect to the Rachev average drawup/down ratio.

4 Conclusion

The fundamental contribution of this paper consists in the methodology to solve dynamic portfolio strategies considering realistic assumptions on the returns. In particular, we examine the impact of simulating a copula with opportune marginals in optimal portfolio choices. We first describe how to generate scenarios that take into account of return anomalies: heavy tailed distributions, volatility clustering, non Gaussian copula dependence. Then, we discuss the use of reward/risk criteria to select optimal portfolios and we propose the use of the STARR ratio and two new dynamic performance measures. Finally, we propose

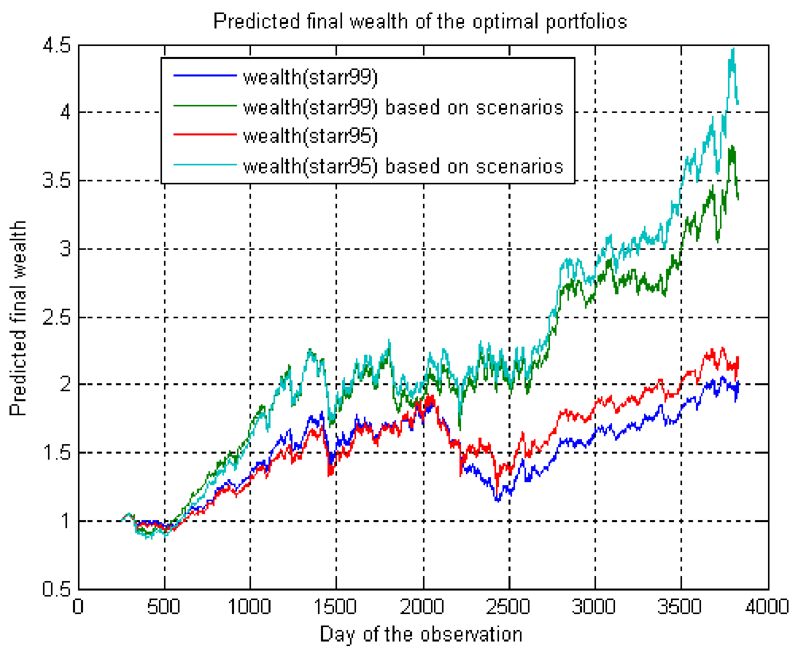

Figure 2: This figure compares the ex-post final wealth obtained maximizing the STARR ratio (with $1-\alpha = 99\%, 95\%$) and using either real data or simulated data.

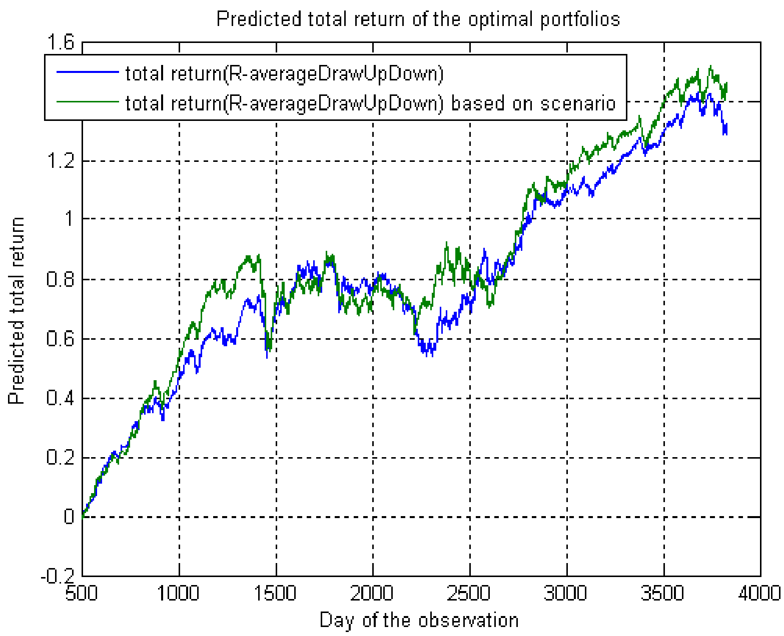

Figure 3: This figure compares the ex-post cumulative return obtained maximizing the R-averagedrawup/down ratio and using either real data or simulated data.

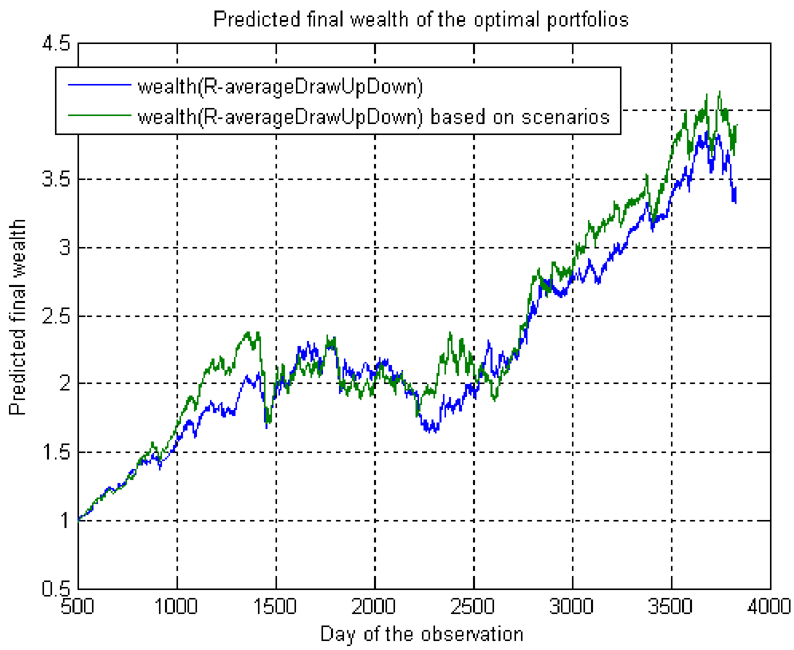

Figure 4: This figure compares the ex-post final wealth obtained maximizing the R-average drawundown ratio and using either real data or simulated data.

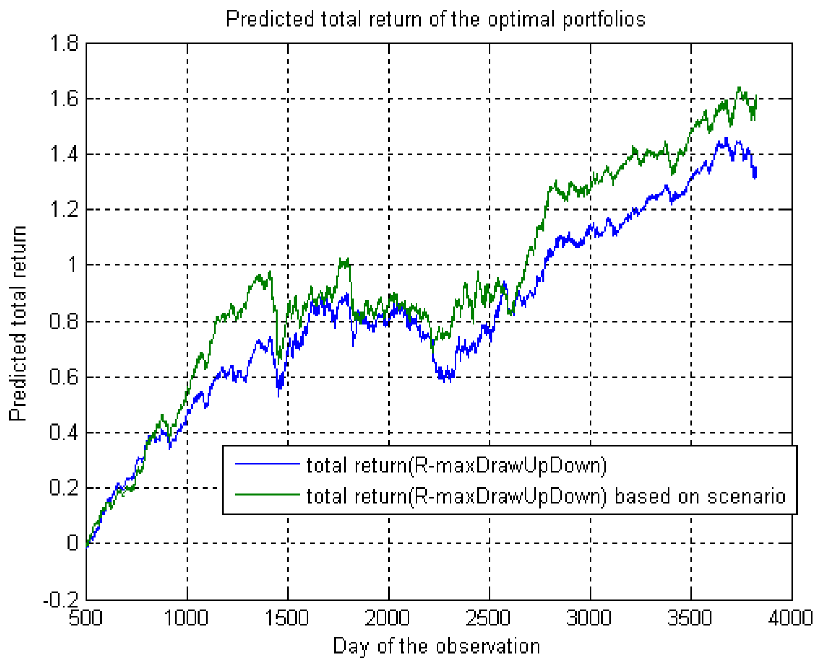

Figure 5: This figure compares the ex-post cumulative return obtained maximizing the R-maximum drawup/down ratio and using either real data or simulated data.

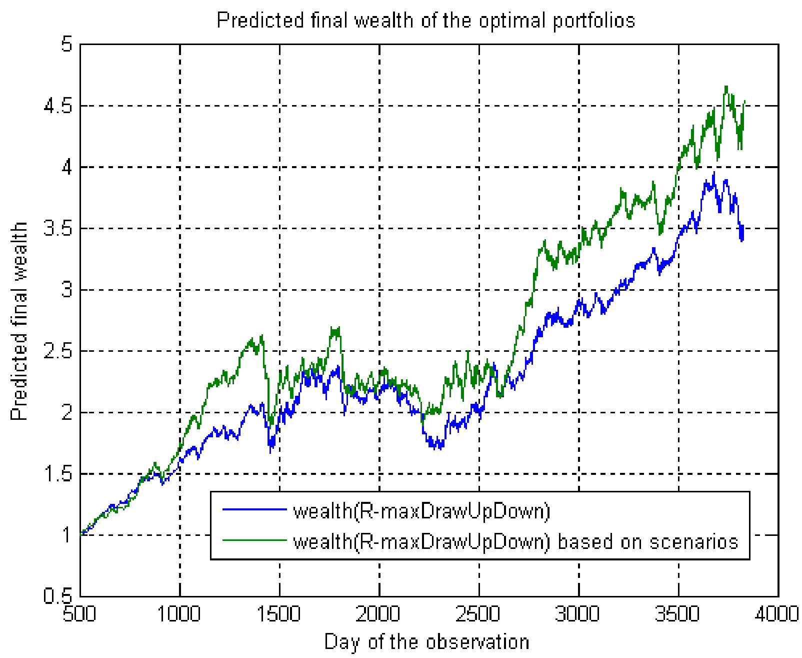

Figure 6: This figure compares the ex-post final wealth obtained maximizing the R-maximum drawup/down ratio and using either real data or simulated data.

an empirical comparison among final wealth and cumulative return processes obtained either using historical observations or using the simulated data. As we expect the ex-post empirical comparison among classic myopic approaches based on historical observations and those based on simulated data shows the greater predictable capacity of the latter.
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