Solutions - 3

Statistical Methods for Financial Risk Management

Exercise 1

We can expect a solution to occur when $\bar{r}'y$ is large and $y'Qy$ is close to $V_a$, where $\rho$ is a factor controlling the balance between return and risk. As in previous exercises, we have

$$\bar{r} = \begin{pmatrix} 1.2833 \\ 1.1167 \end{pmatrix} \quad \quad Q = \begin{pmatrix} 0.0181 & -0.0281 \\ -0.0281 & 0.0514 \end{pmatrix}$$

we write

$$y = \alpha + \beta x$$

where $x = y_1$ and

$$\alpha = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \quad \beta = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$  

We then get the problem

$$\min F = -\bar{r}'\alpha - (\bar{r}'\beta)x + \frac{\rho}{V_a} (\alpha'Q\alpha + 2(\beta'Q\alpha)x + (\beta'Q\beta)x^2 - V_a)^2.$$  

By considering values for $\bar{r}$ and $Q$, we obtain

$$F = -0.0666x - 1.1167 + \frac{\rho}{V_a^2} (0.1256x^2 - 0.1589x + 0.0513 - V_a)^2.$$  

The first derivative is,

$$F'(x) = -0.0666 + 2\frac{\rho}{V_a^2} (0.1256x^2 - 0.1589x + 0.0513 - V_a) (0.2512x - 0.1589),$$

and we can write

$$F'(x) = -0.0666 + 2\frac{\rho}{V_a^2} (0.0316x^3 - 0.0599x^2 + 0.0371x - 0.075).$$

The function is not linear, therefore we have to apply a numerical procedure (bisection, secant or Newton method just to say a few) to find a minimum of $F$. A solution can be found with the Matlab function \textit{fminsearch} or with the equivalent \textit{R} function, $x = 0.7837$. 

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Exercise 2

Data can be downloaded and transform in vector as in a previous exercise, we can call them IBM, JPM, MS. We need only to calculate the variance/covariance matrix, since we are assuming a zero mean value at risk. The variance/covariance matrix is

\[
Q = \begin{pmatrix}
0.0002454501 & 0.0001655138 & 0.0001729856 \\
0.0001655138 & 0.0003924207 & 0.0002672269 \\
0.0001729856 & 0.0002672269 & 0.0003774317
\end{pmatrix},
\]

we can allocate the matrix

\[
Q \leftarrow \text{matrix}(\text{data} = \text{NA}, \text{nrow} = 3, \text{ncol} = 3)
\]

and then

\[
Q[1,1] \leftarrow \text{var}(\text{IBM}) \\
Q[1,2] \leftarrow \text{cov}(\text{IBM}, \text{JPM})
\]

and so on. The standard deviation of the portfolio is \(\sqrt{y^TQy} = 471.8851\), therefore the 10 days zero mean 99% VaR\(^L\) of the entire portfolio is

\[
\text{VaR}^L = 2.33 \times 471.8851 \times \sqrt{10},
\]

that is \(\text{VaR}^L = 3476.9\). There is 1% probability that a normally distributed random variable will decrease in value by more than 2.33 standard deviation. In order to check if there are diversification benefits, we find the 10 days zero mean 99% VaR\(^L\) for each position, that is

\[
\text{VaR}^L_{\text{IBM}} = 1154.35 \\
\text{VaR}^L_{\text{JPM}} = 1459.593 \\
\text{VaR}^L_{\text{MS}} = 1431.446
\]

and thus the value

\[
(1154.35 + 1459.593 + 1431.446) - 3476.9 = 568.489
\]

represents the benefit of the diversification.

For the second part, first we estimate parameters by considering the normal model and the stable model. The package \texttt{fBasics} will be used (remember to load the package). For the normal model we can consider the command

\texttt{nFit(IBM)}
... Estimated Parameter(s):
  mean    sd
-0.0001418076  0.0156606181 and with the command

\[
\text{stableFit(IBM, type = c("q")},
\]
...

Estimated Parameter(s):
  alpha    beta    gamma    delta
 1.4700000000  0.0790000000  0.0072516353 -0.0003990818

we estimate parameters with the McCulloch method or quantile method. The MLE estimation (mle instead of q) seems to be too slow in \textit{R}, but can be also done. To find the quantile, we can use

\[
\text{stableVaR} \leftarrow -\text{qstable}(0.01, 1.47, 0.079, 0.0073, -0.0003990818)
\]

and

\[
\text{normalVaR} \leftarrow -\text{qnorm}(0.01, -0.0001418076, 0.0156)
\]

we obtain the 99\% stable VaR 0.057 and in the same way the normal VaR 0.036. The empirical quantile can be also found

\[
\text{VaR_emp} \leftarrow -\text{quantile(IBM, 0.01)}
\]

and we obtain 0.047. We will call the vector of IBM returns between January 1, 2007 to November 1, 2007, \text{IBMcheck}. We have the following number of data

\[
\text{numdata} \leftarrow \text{length(IBMcheck)}
\]

and we find the number of exceedings

\[
\text{stableEx} \leftarrow \text{sum(IBMcheck<(-stableVaR))}
\]

In both cases we obtain 0. Look at the plots \text{plot(IBM)} and \text{plot(IBMcheck)} to understand the reason. If we estimate parameters only by using the last year of data (250 trading days), we obtain 10 (10/210=4.7\%) exceedings in the normal case and 4 (4/210=1.9\%) in the stable one.