Exercises - 1

Statistical Methods for Financial Risk Management

1. Let $X_1, \ldots, X_n$ be an i.i.d. sequence from $N(\mu, \sigma^2)$ with an unknown set of parameters $\theta = (\mu, \sigma^2)$ and $n \geq 2$.
   - Write the log-likelihood function.
   - Find the maximum likelihood estimation for $\mu$ and $\sigma$.

2. Let $X_1, \ldots, X_n$ be an i.i.d. sequence from $\Gamma(\alpha, \gamma)$ with an unknown set of parameters $\theta = (\alpha, \gamma)$ and $n \geq 2$.
   - Write the log-likelihood function.
   - Find the maximum likelihood estimation for $\gamma$ and describe a procedure to find $\alpha$.

   - Suppose that log returns are normal distributed. Estimate mean and variance.
   - Draw the qq-plot.
   - Find empirical mean, variance, skewness and kurtosis and compare them with the normal distribution obtained in the previous point.
   - Evaluate KS, AD and JB statistics.
   - Find a way to calculate the p-value in the KS Goodness of Fit Test.
   - How can we find the p-value of the AD Goodness of Fit Test?
   - Are log return normal distributed?

4. Describe a procedure to simulate compound Poisson processes. By considering the definition of standard Brownian motion, simulate a trajectory with $t \in [0, 1]$ and by taking time steps $\Delta t = 1/250$.

Some exercises can be solved with paper and pen, other have to be solved with the help of a PC. The free software $R$, http://www.r-project.org, is a good choice for our goal. For any problem or remark, do not hesitate to contact me, bianchi@statistik.uni-karlsruhe.de