The Application of Pairs Trading to Energy Futures Markets

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ABSTRACT

This paper investigates the usefulness of a hedge fund trading strategy known as "pairs trading" applied to energy futures markets. The profit of a simplified pairs trading strategy is modeled by using a mean-reverting process of the futures price spread. According to the comparative statics of the model, the strong mean-reversion and high volatility of the spread give rise to the high expected return from trading. Analyzing energy futures (more specifically, WTI crude oil, heating oil, and natural gas futures) traded on the New York Mercantile Exchange, we present empirical evidence that pairs trading can produce a relatively stable profit. We also identify the sources of the profit from a pair trading strategy, focusing on the characteristics of energy. The results suggest that natural gas futures trading may be more profitable than WTI crude oil and heating oil due to its strong mean-reversion and high volatility, while seasonality may also be relevant to the strategy's profitability. Moreover, natural gas futures trading may be more vulnerable to event risk (as measured by price spikes) than the others. Finally, we investigate the profitability of cross commodities pairs trading.

Key words: Pairs trading, energy futures markets, futures price spread, dynamic conditional correlation model, WTI crude oil, heating oil, natural gas
JEL Classification: C51, G29, Q40

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1. Introduction

One well-known trading strategy employed by hedge funds is “pairs trading” and it is typically applied to common stock. This trading strategy is categorized as a statistical arbitrage and convergence trading strategy. In pairs trading, the trader identifies two brands of stock prices that are highly correlated based on their price histories and then starts the trades by opening the long and short positions of the two brands selected. Gatev, Goetzmann, and Rouwenhorst (2006) conduct empirical tests of pairs trading from common stock. They show that a pairs trading strategy is profitable, even after taking into account transaction costs. Jurek and Yang (2007) compare the performance of their optimal mean-reversion strategy with that of Gatev, Goetzmann, and Rouwenhorst (2006) using simulated data. They show that their strategy provides even better performance than that of Gatev, Goetzmann, and Rouwenhorst. While a pairs trading strategy has been applied primarily as a stock market trading strategy, there is no need to limit the strategy to that asset class. A pairs trading strategy simply requires two highly correlated prices. To the best of our knowledge, a pairs trading has not been investigated for energy markets. A priori, there is reason to expect that pairs trading might be successful in energy markets: As reported by Alexander (1999), energy futures with different maturities are characterized by highly correlated prices.

In attempting to apply pair trading as used in the stock market to the energy market, care must be exerted. Energy prices exhibit unique characteristics compared to stocks. These characteristics include strong seasonality, strong mean-reversion, high volatility, and sudden price spikes (see Eydeland and Wolyniec (2003)). Because these characteristics may have a positive or negative impact on a pairs trading strategy, in this paper we investigate the impact of these characteristics on the profitability of a pairs trading strategy.

In order to examine the profitability of pairs trading applied to energy markets, we propose a simplified profit model for pairs trading, focusing on how the convergence and divergence of pairs generate profits and losses. We begin by modeling the price spread of energy futures using a mean-reverting process as in Dempster, Medova, and Tang (2008). Then, we obtain the profit model by calculating the total gain and loss stemming both from the profits of spread convergence during the trading period and from the losses of spread divergence at the end of the period. In addition, we examine the comparative statics of total profit by changing the mean-reversion strength and the volatility of the spread. We show that the total profit is positively correlated with the mean-reversion spread and the volatility of the spread.

We conduct empirical analyses of pairs trading by using energy futures prices for WTI crude oil, heating oil, and natural gas traded on the New York Mercantile Exchange (NYMEX). We find that pairs trading can produce relatively stable profits for all three energy markets. We then investigate the principal factors impacting the total profits, focusing on the characteristics of energy: strong seasonality, strong mean-reversion, high volatility, and large price spikes. Since we observe that seasonality for heating oil and natural gas affects the strategies profitability, the total profits may be determined by taking into account seasonality. Then, we find that mean-reversion and volatility appear to be a key to the profitability of the trading strategy. In addition, we examine event risk in pairs trading, as gauged by price spikes, by analyzing the correlation between the pairs obtained from the dynamic conditional correlation model (see Engle (2002)). Our results suggest that natural gas futures trading may be more vulnerable to event risk as measured by price spikes than WTI crude oil and heating oil trading. Finally, we investigate the profitability of cross commodity trading that employs all three energy futures prices.

This paper is organized as follows. Section 2 proposes a profit model for a simplified pairs trading strategy. Section 3 explains how the model is modified for energy futures. Section 4 analyzes the usefulness of pairs trading in energy futures markets; our conclusions are summarized in Section 5.

2. The Profit Model

As explained earlier, in general, pairs trading involves the simultaneous purchase of one financial instrument and sale of another financial instrument with the objective of profiting from not the movement of the absolute value of two prices, but the movement of the spread of the prices. The basic and most common case of pairs trading involves simultaneously selling short one financial instrument with a relatively high price and buying one financial instrument with a relatively low price at the inception of the trading period, expecting that in the future the higher one will decline while the lower one will rise. If the two prices converge during the trading period, the position is closed at the time of the convergence. Otherwise, the position is forced to close at the end of the trading period. For example, suppose we denote by $P_{1,t}$ and $P_{2,t}$ the relatively high and low financial instrument prices respectively at time t during the trading period. When the convergence of price spread occurs at time τ during the trading period, the profit is the price spread at time 0 denoted by $x = P_{1,0} - P_{2,0}$. In contrast, when the price spread does not converge until the end of the trading period at time T, the profit stems from the difference between the spreads at times 0 and T as $x - y = (P_{1,0} - P_{2,0}) - (P_{1,T} - P_{2,T})$, where y represents the price spread at time T. Thus, pairs trading produces a profit or loss from the relative price movement, not the absolute price movement.[1]

[1]The position will be scaled such that the strategy will be self-financing. Even in this case, the price spread is relevant to the profit.

In this section, we present a profit model for this basic and most common pairs trading strategy. To do so, we need to take into account in the model (1) the price spread movement and (2) the frequency of the convergence. We utilize these modeling components to derive our profit model in order to formulate a general model that can be applied to pairs trading regardless of the financial instrument.

Let's look first at the modeling of the price spread movement. As a first-order approximation, we assume that the price spread of a pairs trade, denoted by S_t, follows a mean-reverting process given by

$$dS_t = \kappa(\theta - S_t)dt + \sigma dW_t. \tag{1}$$

For simplicity, we assume that κ, θ, and σ are constant. Equation (1) was used by Jurek and Yang (2007) as a general formulation for an investor facing a mean-reverting arbitrage opportunity and by Dempster, Medova, and Tang (2008) for valuing and hedging spread options on two commodity prices that are assumed to be cointegrated in the long run.

Next, we need to capture how frequently the price spread converges. Assume that a sample path of a stochastic process with an initial value X at time 0 reaches Y at time τ for the first time. τ is refereed to as the "first hitting time." Linetsky (2004) provides an explicit analytical characterization for the first hitting time (probability) density for a mean-reverting process that can be applied to modeling interest rates, credit spreads, stochastic volatility, and convenience yields. Here we apply it to modeling pair trading because it occurs to us that the behavior such that X reaches Y for the first time can help to express the convergence of price (i.e., price spread X at time 0 converges on $Y = 0$ at time τ.). Moreover, we believe that the correponding probability density is useful to calculate the expected profit from the pairs trading strategy.

Finally, we calculate the profit from pairs trading using a mean-reverting price spread model and first hitting time density for a mean-reverting process. This calculation takes into account the convergence and failure to converge of the price spread during the trading period. First, consider the case of convergence (i.e., when price spread convergence occurs which is responsible for producing the strategy's profit). In this case, the expected profit, denoted by $r_{p,c}$, is calculated as the product of price spread x at time 0 and the probability for price spread convergence until the end of the trading period, because x is given and fixed as an initial value. Since the spread becomes 0 from the initial value $x > 0$, the first hitting time density for a mean-reverting process in Linetsky (2004) is employed. Now let's look at the case when the price spread reaches y at the end of the trade, failing to converge during the trading period. The expected profit, denoted by $r_{p,nc}$, is calculated as the integral (with respect to y) of the product of $x - y$ and the probability density for failure to converge during trading period, resulting in the arrival at y at the end of the trading period, where the probability is again obtained from the first hitting time density for a mean-reverting process in Linetsky (2004).

Given the above, we can now derive our profit model, denoted by r_p , from a simple pairs trading strategy. Suppose that x is the price spread of two assets used for pairs trading at time 0 when trading begins by constructing the long and short positions. Suppose further that the price spread is given by a mean-reverting model as in equation (1).

First, let's consider the spread convergence case in which price spreads at times 0 and τ are fixed as x and 0, respectively. Assuming that the first hitting time density for the price spread process is $f_{\tau_{x\to 0}}(t)$, the expected return $r_{p,c}$ for this spread convergence is represented as the expectation value:

$$r_{p,c} = x \int_0^T f_{\tau_{x\to 0}}(t)dt. \tag{2}$$

We assumed the process follows a mean-reverting process. By Proposition 2 in Linetsky (2004), the first hitting time density of a mean-reverting process moving from x to 0 is approximately:

$$f_{\tau_{x\to 0}}(t) \simeq \sum_{n=1}^{\infty} c_n \lambda_n e^{-\lambda_n t}, \tag{3}$$

where λ_n and c_n have the following large-n asymptotics:

$$\begin{aligned}
\lambda_n &= \kappa\left\{2k_n - \frac{1}{2}\right\}, \\
c_n &\simeq \frac{(-1)^{n+1} 2\sqrt{k_n}}{(2k_n - \frac{1}{2})(\pi\sqrt{k_n} - 2^{-\frac{1}{2}}\bar{y})} e^{\frac{1}{4}(\bar{x}^2 - \bar{y}^2)} \cos\left(\bar{x}\sqrt{2k_n} - \pi k_n + \frac{\pi}{4}\right), \\
k_n &\simeq n - \frac{1}{4} + \frac{\bar{y}^2}{\pi^2} + \frac{\bar{y}\sqrt{2}}{\pi}\sqrt{n - \frac{1}{4} + \frac{\bar{y}^2}{2\pi^2}}.
\end{aligned}$$

$\bar{x}$ and $\bar{y}$ are expressed by

$$\bar{x} = \frac{\sqrt{2\kappa}}{\sigma}(x - \theta) \text{ and } \bar{y} = \frac{\sqrt{2\kappa}}{\sigma}(-\theta).$$

Thus, the profit model for the convergence case is modeled by

$$r_{p,c} \simeq x \sum_{n=1}^{\infty} c_n \left(1 - e^{-\lambda_n T}\right). \tag{4}$$

Next consider the case where there is a failure to converge during the trading period. This case holds if the price spread does not converge on 0 during the trading period and then reaches any price spread y at the end of the trading period. Note that y is greater than or equal to 0, otherwise, the process converges on 0 before the end of the trading period. The event for this case is represented by the difference between two events: (1) when the process with initial value of x at time 0 reaches y at time T and (2) when it arrives at y at time T after the process with initial value of x at time 0 converges on 0 at any time t during the trading period. We denote the corresponding distribution functions by $g(y;x,T)$ and $k(y;x,T)$, respectively. In addition, the payoff of the trading strategy is given by $x-y$. A profit model for pairs trading due to the failure to converge is expressed by the expectation value:

$$r_{p,nc} = \int_0^\infty (x-y)\{g(y;x,T) - k(y;x,T)\}dy. \tag{5}$$

The probability density for the latter event, $k(y;x,T)$, is represented by the product of the first hitting time density $f_{\tau_{x\to 0}}(t)$ and the density $g(y;0,T-t)$, meaning that the process reaches y after the first touch because both events occur independently. Since t can be taken as any value during the trading period, the density function $k(y;x,T)$ is calculated as the integral of the product with respect to t, as given by

$$k(y;x,T) = \int_0^T f_{\tau_{x\to 0}}(t)g(y;0,T-t)dt. \tag{6}$$

A profit from pairs trading for the failure to converge case is

$$r_{p,nc} = \int_0^\infty (x-y)\left\{g(y;x,T) - \int_0^T f_{\tau_{x\to 0}}(t)g(y;0,T-t)dt\right\}dy. \tag{7}$$

Because the price spread S_t follows a mean-reverting process, the corresponding probability density is given by

$$g(y;x,T) = \frac{1}{\sqrt{2\pi}\sigma_S(T)}e^{-\frac{1}{2}\frac{(y-\mu_S(x,T))^2}{\sigma_S(T)^2}}, \tag{8}$$

where

$$\mu_S(x,T) = xe^{-\kappa T} + \theta(1-e^{-\kappa T}) \text{ and } \sigma_S(T) = \sqrt{\tfrac{\sigma^2}{2\kappa}(1-e^{-2\kappa T})}.$$

In addition, recall that the first hitting time density for a mean-reverting process is given by equation (3). The expected profit from the failure to converge case is approximately modeled as follows:

$$r_{p,nc} \simeq \int_0^\infty (x-y)\left\{g(y;x,T) - \int_0^T \sum_{n=1}^\infty c_n\lambda_n e^{-\lambda_n t} g(y;0,T-t)dt\right\}dy. \tag{9}$$

Thus, the total profit model for pairs trading is expressed by

$$r_p = r_{p,c} + r_{p,nc}, \tag{10}$$

where $r_{p,c}$ and $r_{p,nc}$ are calculated from equations (4) and (9), respectively.

Since both $\bar{x}$ and $\bar{y}$ embedded in the coefficients of equations (4) and (9) are expressed by functions of the mean-reversion strength and volatility of the spread denoted by κ and σ, respectively in equation (1), the expected profit from a simple pairs trading strategy is influenced by the degree of mean-reversion and volatility. More precisely, we have formulated the expected return from spread convergence and failure to converge cases using a simple mean-reverting spread model characterized by κ and σ. The model is general in that it does not specify the financial instrument. The requirement is that price spread follows a mean-reverting process. In addition, the model expresses a basic and most common case for pairs trading in that if price spread converges during the trading period, the profit is recognized, otherwise, the loss or profit is recognized at the end of the trading period.

While the profit model presented in this section applies to any financial instrument, there are nuisances when seeking to apply the model to specific sectors of the financial market that require the model be modified. In Section 3, we apply pairs trading to energy futures and explain how the profit model must be modified.

3. Application of the Profit Model to Energy Futures Markets

3.1. The profit model for energy futures

Recall that the requirement to use the profit model proposed in the previous section was just that the price spread process follows a mean-reverting process in equation (1). If price spreads of energy futures mean revert, the profit model can be directly applicable to energy futures. We will empirically examine the mean-reversion issue for energy futures here. Figure 1 shows the term structure of the NYMEX natural gas futures, which comprise six delivery months (shown on the maturity axis) and whose daily observations cover from April 3, 2000 to March 31, 2008 (shown on the time axis). As can be seen, the shape of the futures curve with six delivery months fluctuates as time goes on during the period.

A closer look at Figure 1 illustrates that the futures curve sometimes exhibits backwardation (e.g., April 2002) and at other times contango (e.g., April 2004).[2] The existence of backwardation and contango at different moments gives rise to the existence of the flat term structure that switches from backwardation to contango, or vice versa. Thus, energy futures prices that have different maturities converge on the flat term structure, even though price deviates from the others owing to the backwardation or contango. These attributes that we observe for the term structure of energy futures prices may be useful in obtaining the potential profit in energy futures markets using the convergence of the price spread between the two correlated assets as in pairs trading. More importantly, the transition of price spreads from backwardation or contango to the flat term structure can be considered as a mean-reversion of the spread.

[INSERT FIGURE 1 ABOUT HERE]

In order to support this conjecture, we estimate the following autoregressive 1 lag (AR(1)) model for price spreads ($P^i - P^j$) for i and j month natural gas futures ($i \neq j$).

$$P_t^i - P_t^j = C_{ij} + \rho_{ij}(P_{t-1}^i - P_{t-1}^j) + \varepsilon_t, \tag{11}$$

where $\varepsilon_t \sim N(0, \sigma_{i,j}^2)$. The results are reported in Table 1.

[INSERT TABLE 1 ABOUT HERE]

As can be seen from the table, AR(1) coefficients for all combinations of the price spreads are statistically significant and greater than 0 and less than 1. Given the characteristics of the price spreads for energy futures that we observe, we conclude that a mean-reverting model can be applicable to energy futures price spreads. In fact, as noted by Dempster, Medova, and Tang (2008), price spreads for energy futures are modeled by a mean-reverting process. Thus, the profit model for energy futures is directly employed as the model in equation (10) without any modification.[3]

3.2. Comparative statics of expected return

Since the profit model is based on a mean-reverting spread model, the expected returns are characterized by the strength of mean-reversion and volatility. Considering that strong mean-reversion

[2] When the future curve increases in maturity, it is called “contango”. When it decreases in maturity, it is called “backwardation”.

[3] The model may be extended so as to capture the non-Gaussian distribution of the spread and the volatility clustering, but we employ a mean-reverting model for simplicity in order to observe the impact of mean reversion and volatility on the profit from pairs trades in energy futures markets in Section 3.2 .

and high volatility in prices are often observed in energy futures markets, it may be significant to evaluate the influence of both characteristics on the profitability of the trade using the profit model for energy futures. We conducted comparative statics of the expected return using the simple profit model we propose in equation (10). As a base case, we used the following values for the parameters of equation (1): $\kappa = 0.027,\ \ \theta = 0,\ $ and $\ \sigma = 0.013$. Moreover, we assumed that the maturity is 120 days and $x = 2\sigma$. Based on the estimated parameters and the assumptions made, we calculated the expected return for the simple profit model (r_p) to be 0.0140.[4] Thus, the model predicts that the initial spread of $x = 2\sigma = 0.0260$ will produce a profit of 0.0140 from pairs trading during a 120-day trading period.

We calculated the expected returns by changing the mean-reversion strength κ and the volatility σ, respectively, in a way that one is changed and the other is fixed. First, we only increase the mean-reversion κ from 0.024 to 0.027. As can be seen in Figure 2, the corresponding expected return increases from 0.0119 to 0.0140. This suggests that higher mean-reversion produces a higher expected return for the model. This is consistent with intuition: If mean-reversion of the spread is stronger, the spread converges more immediately and the convergence generates more profit.

[INSERT FIGURE 2 ABOUT HERE]

Second, we increase the volatility σ from 0.0112 to 0.0130, holding mean-reversion constant at 0.027. Figure 3 shows that the return increases from 0.0114 to 0.0140. This result suggests that higher spread volatility produces higher expected return. This is also consistent with intuition: If spread volatility is higher, the chance of spread convergence increases, resulting in a greater return. Thus, this simple comparative analysis of the influence of spread behavior on the profits suggests that stronger mean-reversion and higher spread volatility may produce higher profit from a pairs trading strategy. Recognizing that strong mean-reversion and high spread volatility are often observed in energy markets, this suggests that a pairs trading strategy applied to this market may be profitable.

[INSERT FIGURE 3 ABOUT HERE]

In the next section, we present our empirical findings on the pairs trading strategy applied to the NYMEX energy futures market.

[4]We numerically calculated r_p using equation (10) via equations (4) and (9).

4. Empirical Studies for Energy Futures Prices

4.1. Data

In this study, we use the daily closing prices of WTI crude oil (WTI), heating oil (HO), and natural gas (NG) futures traded on the NYMEX. Each futures product includes six delivery months – from one month to six months. The time period covered is from April 3, 2000 to March 31, 2008. The data are obtained from Bloomberg. Summary statistics for WTI, HO, and NG futures prices are provided in Tables 2, 3, and 4, respectively.[5,6] These tables indicate that WTI, HO, and NG have common skewness characteristics. The skewness of WTI, HO, and NG futures prices is positive, meaning that the distributions are skewed to the right.

[INSERT TABLE 2 ABOUT HERE]

[INSERT TABLE 3 ABOUT HERE]

[INSERT TABLE 4 ABOUT HERE]

4.2. Pairs trading in energy futures markets

Here we will discuss the empirical tests of the pairs trading strategy in the energy markets using the historical prices of energy futures described in Section 4.1. We explain the practical concept of pairs trading using Figure 4 in accordance with Gatev, Goetzmann, and Rouwenhorst (2006). The prices are first normalized with the first day's price as in the figure, which represent the cumulative returns. A pairs trading strategy involves selecting the most correlated pair of prices during the period. This period is referred to as the *formation period.*[7] Specifically, two assets are chosen such that the standard deviation of the price spreads in the formation period is the smallest in all combinations of pairs.[8] Then, the trade is implemented by using the selected pairs during the consecutive period, referred to as the *trading period.* When the price spread of the selected pairs reaches a user specified multiple of the standard deviation calculated in the formation period, a zero-cost portfolio

[5] The kurtosis of the normal distribution is 3.

[6] We denote the maturity month by a subindex. For example, WTI_i represents i month maturity WTI futures.

[7] Since two futures contracts on the same commodity form a "natural pair," one might not think that a formation period is required for pairs trading in energy futures markets. However, there are many combinations of pairs in this yield curve play. It is safe for investors to select the most correlated pairs even for two futures contracts on the same commodity. Thus, we introduce a formation period for energy futures pairs trading.

[8] It corresponds to the selection of the highly correlated pairs in practice.

is constructed. In our study, we specify a multiple of 2. The zero-cost portfolio is constructed such that the higher and lower price assets are set to short and long positions, respectively. If the pairs converge during the trading period, then the positions are closed and the profit is recognized. Otherwise, the positions are forced to close and it may cause a loss from the trade. We then calculate the total profit from the trades by computing the sum of the trading gains and losses.[9]

[INSERT FIGURE 4 ABOUT HERE]

Now we empirically examine the profitability of pairs trading in energy futures markets of WTI, HO, and NG stemming from the convergence of the different maturity futures price spreads. We discuss the source of the profitability taking into account characteristics of energy prices: seasonality, mean-reversion, and volatility.[10] Following Gatev, Goetzmann, and Rouwenhorst (2006), we set both the formation period and trading period at 120 trading days. Fifteen pairs are chosen during the formation period based on the small standard deviation of the spreads. Then, we set 88 formation and trading periods whose starting points are determined by rolling over 20 trading days in an effort to exploit the available data effectively.[11] We report the annualized profits from the WTI, HO, and NG futures trades in Figure 5. The results suggest that each trade tends to generate a profit except around trade #25 whose trading period starts from September 3, 2002 to February 25, 2003. These findings suggest that pairs trading in WTI, HO, and NG futures markets may be profitable.

[INSERT FIGURE 5 ABOUT HERE]

Having established that a pairs trading strategy is profitable, we next turn our investigation to the source of the profitability in terms of the characteristics of energy. In order to examine the influence of seasonality, mean-reversion, and volatility in energy prices on the total profit, we summarized all the trades in Tables 5 and 6. Note that we refer to the total profit as the excess return because the trading gains and losses are calculated over long and short positions of one dollar and they are considered as the excess returns on the undervalue above overvalue futures as in Gatev, Goetzmann, and Rouwenhorst (2006).

[INSERT TABLE 5 ABOUT HERE]

[9] Here the pairs trading allows multiple trades during the trading period as in Figure 4.
[10] Price spikes are investigated in Section 4.3.
[11] If the first trade starts on the first trading day, the second one starts on the 21st trading day.

[INSERT TABLE 6 ABOUT HERE]

First, we address the influence of seasonality of energy futures prices on the profit generated from pairs trading by focusing on winter. To do this using a simple examination, we define "winter trade" presented in Tables 5 and 6 by the trade including the days from December to February. In Tables 5 and 6, the winter trades comprise eight periods: the trades from #1 to #4, from #12 to #16, from #25 to #29, from #37 to #41, #50 to #54, #62 to #66, #75 to #79, and #87 to #88.

Table 5 shows that all winter trades of WTI, HO, and NG from #1 to #4 produce profits. From #12 to #16, the trades of HO and NG produce profits, while the trades of WTI have annualized losses of -0.36 and -0.09 for #12 and #13, respectively. The winter trades of WTI, HO, and NG from #26 to #29 produce profits.[12] Accordingly, these findings for the winter trades suggest that while the WTI trades produce both gains and losses in the winter, the HO and NG trades always make profits in the winter. In addition, the trades of HO and NG in the summer from #5 to #8 and from #17 to #19 produce low profits or losses while the WTI trades generate profits to some extent. The same characteristics explained above are observed in the other winter trades in Tables 5 and 6. While the profits from the WTI trades are not relevant to the season, seasonality in profitability from HO and NG trades exists in the sense that the trades of HO and NG in the winter tend to cause high profits while those in the summer show low profits or losses. The results correspond to the existence of seasonality explained in Pilipovic (1998) such that HO and NG futures prices exhibit seasonality contrary to WTI futures prices.

Since we are interested in the influence of seasonality on the total profit from energy futures pairs trading, we compared the annualized average profits between WTI, HO, and NG. According to the results reported in Table 6, the profits from six WTI, HO, and NG futures trades are 0.61, 0.74, and 3.15, respectively. We found that the NG trading generates the largest total profits (i.e., the highest excess return) of the three contracts investigated during the trading periods as in Table 6, while the HO and the WTI showed the second and the third highest excess return, respectively. Since the profits from the WTI trades without seasonality are less than those from the HO and NG trades with seasonality, this factor may determine the total profit.

Second, we can shed light on the mean-reversion and volatility of the spread as the source of the total profits from pairs trading. For all combinations of the spreads for each energy futures, we estimate the parameters of AR(1) coefficient ρ and standard deviation σ, which represent mean-reversion strength and volatility, respectively, using equation (11). Then, we take the average $\bar{\rho}$ and $\bar{\sigma}$ of the estimated ρ's and σ's, respectively, in order to capture the average trend of the mean-reversion and volatility of each energy futures market. The results are shown in Table 7. As can

[12] Later in this paper we discuss trade #25 as a special case.

be seen, the price spread of NG futures has the strongest mean-reversion and the highest volatility of the three, because the AR(1) coefficient is the smallest (0.981) and the standard deviation is the largest (0.257) of the three. Considering that the NG trades generate the highest average return (3.15) of the three, the highest total profit of pairs trading applied to NG futures may come from the strongest mean reversion and highest volatility of the spread. The comparative statics in Section 3.2 suggested that the strong mean-reversion and high volatility of price spreads may produce high profit from pairs trading. Thus, the results obtained in this section may also be supported by the results from the comparative statics.

[INSERT TABLE 7 ABOUT HERE]

As is discussed, the results from the empirical analyses document that strong mean-reversion and high volatility of price spreads, as well as seasonality, may influence the total profit from pairs trading of energy futures.

4.3. Pairs Trading and Event Risk

When we studied the influence of seasonality on the profit from pairs trading in the previous section, trade #25 was not relevant to the seasonality-based profit in the sense that the HO and NG trades exhibited huge losses in spite of winter trades. In this section, we attempt to explain the reason for the exception by exploring the influence of event risk as measured by price spikes on the profit from pairs trading.

We begin by examining the relationship between the risk and return for the trading strategy, since price spikes may affect the relationship. We illustrate the return distributions of pairs trading for three energy futures in Figure 6. The WTI trading achieves sustainable profits with small losses as shown in the top part of Figure 6, because the distribution almost lies in the positive region and the width of the distribution is the smallest of the three futures. It corresponds to the positive mean (0.61) and the smallest variance (1.03) in Table 6. In contrast, the NG futures trading produces not only large profits but also a large loss (-4.35) as shown in the bottom part of Figure 6, leading to the largest variance (8.52) in Table 6. HO trading is in the middle in terms of the risk and return relationship as shown in the middle part of Figure 6.

[INSERT FIGURE 6 ABOUT HERE]

In addition, we examined fat-tailed and skewed distributions for the returns by estimating the well-known α-stable distribution parameters of α, β, μ, and σ. The parameter α describes the kurtosis of the distribution with $0 < \alpha \leq 2$. The smaller the α, the heavier the tail of the distribution. The parameter β describes the skewness of the distribution, $-1 \leq \beta \leq 1$. If β is positive (negative), then the distribution is skewed to the right (left). μ and σ are the shift and scale parameters, respectively. If α and β equal 2 and 0, respectively, then the α-stable distribution reduces to the normal one. The estimation results are reported in Table 8. Since αs for WTI and NG are 2, both return distributions do not have fat tails. On the other hand, since α for HO is 1.464, the return distribution has a fat tail. Then, since βs for WTI, HO, and NG are 1, the distributions are all skewed to the right. Thus, the returns to pairs trades in energy futures markets may more or less be influenced by infrequent extreme loss, resulting in what economists refer to as the "peso problem," observed by Milton Friedman about the Mexican peso market of the early 1970s.

[INSERT TABLE 8 ABOUT HERE]

We next investigate the source of this huge loss by analyzing the price behavior of the pairs in detail. The huge loss in NG futures trading occurs in trade #25 in Table 5, while the loss is included in winter trades that are expected to produce profits due to seasonality. We illustrate the price movement of a typical pair for NG #25 trading in Figure 7 which shows the total returns of NG 1- and 4-month futures by lines without markers and with circle markers, respectively. The total return for the trading strategy is shown by a line with square markers. If the trade is open, the line with square markers is described as the upper stair and otherwise, it is described as the lower stair. According to Figure 7, after the position opens on December 11, 2002 due to the expansion of price spread by more than two standard deviation in the formation period, the spread expands dramatically just before the breakout of the Iraq war around the end of February 2003. The trading position is forced to close without convergence at the end of the trading period. It gives rise to the large loss from pairs trading. Therefore, the event risk expressed as price spikes may dramatically adversely impact the relatively stable profit from NG futures pairs trading.

[INSERT FIGURE 7 ABOUT HERE]

In order to support the above observation of the impact of event risk, we will capture the correlation structure of energy futures prices by using the dynamic conditional correlation (DCC) model for prices p_t (see Engle (2002)):[13]

$$p_t = \phi_0 + \phi_1 p_{t-1} + \varepsilon_t, \tag{12}$$

$$\varepsilon_t = D_t \eta_t, \tag{13}$$

$$D_t = \mathrm{diag}[h_{1t}^{\frac{1}{2}}\ h_{2t}^{\frac{1}{2}}], \tag{14}$$

where $p_t = [p_{1,t}\ p_{2,t}]'$, $\phi_0 = [\phi_{1,0}\ \phi_{2,0}]'$, and $\phi_1 = [\phi_{1,1}\ \phi_{2,1}]'$.[14] For $i = 1,\ 2$, we have

$$h_{it} = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{i,t-1}, \tag{15}$$

$$E[\varepsilon_t \varepsilon_t' \mid F_{t-1}] = D_t R_t D_t, \tag{16}$$

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1}, \tag{17}$$

$$Q_t = (1 - \theta_1 - \theta_2) Q + \theta_1 \eta_{t-1} \eta_{t-1}' + \theta_2 Q_{t-1}, \tag{18}$$

where Q_t^* is the diagonal component of the square root of the diagonal elements of Q_t.[15] Here, h_{it}s $(i = 1, 2)$ represent the GARCH(1,1) model for pair prices, respectively. The covariance matrix Q_t is represented by the past noise η_{t-1} and the past covariance matrix Q_{t-1}. If either of the estimates of θ_1 or θ_2 in equation (18) is statistically significant, the correlation R_t of the pairs becomes time-varying.

We estimated the parameters of the DCC model and then calculated the conditional correlations, employing 1- and 2-month maturity futures prices of the WTI, HO, and NG using the sample data from April 3, 2000 to July 10, 2003 which covers trade #25.[16] After the estimation of the AR(1) model, we estimated the DCC model. The estimates of the parameters of the AR(1) model for WTI, HO, and NG futures prices are reported in Table 9. Since all AR(1) parameters in the table are statistically significant, there exist AR(1) effects both in the prices of 1- and 2-month maturity futures prices for the three futures contracts. Next, we estimated the parameters of

[13] For pairs trading, since the correlations between price levels generate the profit, we examine the correlations between prices assuming that the prices are mean reverting. However, the appendix to this paper also examines the correlations by using price returns and shows that the same results as reported in this section are obtained from the price returns.

[14] It may be possible that the sample distribution of the innovations η_t is not normally distributed; however for simplicity we assume that it is normally distributed and independent.

[15] Define $Q_t \equiv \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix}$. Then, $Q_t^* = \begin{pmatrix} \sqrt{q_{11}} & 0 \\ 0 & \sqrt{q_{22}} \end{pmatrix}$.

[16] We select 1- and 2-month maturity pairs in the sense that the spread may explicitly reflect the impact of event risk since the pairs may strongly move together due to the Samuelson effect such that the standard deviation of futures price returns decreases as the maturity increases (see Samuelson (1965)). The effect is especially pronounced in energy markets.

the DCC model. The parameter estimation results and the conditional correlations for WTI, HO, and NG are given in Table 10 and Figure 8, Table 11 and Figure 9, and Table 12 and Figure 10, respectively.

[INSERT TABLE 9 ABOUT HERE]

[INSERT TABLE 10 ABOUT HERE]

[INSERT FIGURE 8 ABOUT HERE]

[INSERT TABLE 11 ABOUT HERE]

[INSERT FIGURE 9 ABOUT HERE]

[INSERT TABLE 12 ABOUT HERE]

[INSERT FIGURE 10 ABOUT HERE]

According to the standard errors of θ_1s and θ_2s for WTI, HO, and NG as in Tables 10, 11, and 12, respectively, the conditional correlations of HO and NG are time-varying because the θ_1, θ_2, or both are statistically significant. Judging from Figures 8, 9, and 10, the correlation of WTI futures is high and stable, that of HO futures is volatile, and that of NG futures is high and stable except for the first quarter 2003. By using these results, we discuss the relationship between the correlations and event risk, where the Iraq war breaks out around the first three months of 2003. Figure 8 for WTI suggests that event risk does not seem to influence the correlations of the WTI futures prices. Figure 9 for HO shows that event risk may more or less influence the correlations, while we cannot distinguish the influence of event risk from the volatile correlations observed during the entire trading period. However, Figure 10 for NG suggests that event risk strongly affects the correlations, resulting in a small correlation (0.45) in February 2003. Judging from the analyses using the DCC model, the trades of NG futures are the most vulnerable to event risk as measured by price spikes, corresponding to the results in Table 6.[17]

Event risk captured as price spikes as in Figures 7 and 10 have a strong impact on the profit from NG pairs trading. Thus the price spikes, one of the most well-known characteristics of energy

[17] We also estimated the DCC model for the three energy markets and obtained the conditional correlation using not the price model in equation (12) but log price return model in equation (A1) of the appendix to this paper. The results are almost identical as the characteristics of the correlations obtained in this section.

prices, may also lead to the strong deterioration of the total profit from pairs trading which is highlighted in the natural gas futures trades, depending on the magnitude and frequency of the spikes.

4.4. Cross Commodities Pairs Trading

Finally, we investigate the profitability of cross commodities pairs trading that can also make use of different commodity pairs such as 1-month NG and 6-month WTI futures in comparison to the pairs trading of single commodity futures examined in Section 4.2. Here, we refer to the improvement of profitability by cross commodities as the "portfolio effect," while we refer to the advantage in the profitability by single commodity as the "term structure convergence effect." If the total profit from cross commodities trading exceeds that from the sum of the three commodity pairs trading in Table 6, the portfolio effect can improve the profits in pairs trading more than the term structure convergence effect. We employ all combinations of WTI, HO, and NG futures prices in Section 4.2 and choose the highly correlated 45 pairs in order to attain equal footing of the profits from all the commodity trades in Section 4.2.[18]

The plots in Figure 11 show that cross commodity trading generated losses in the consecutive periods from trades #22 to #29 and from trades #54 to #61, while it almost produces profits from trades #1 to #88 except for several trades that are almost equal zero such as trade #2. This is different from single commodity pairs trading of NG in Figure 5 because the NG trades also generate profits from the trades from #22 to #29 and from #54 to #61.

[INSERT FIGURE 11 ABOUT HERE]

We then computed the total average profit of cross commodity pairs trading to be 3.04, while the sum of single commodity trading profit from WTI, HO, and NG to be 4.50 as in Table 6. Our findings suggest that the portfolio effect due to the cross commodity using three energy futures may not improve the total profit of the pairs trading strategy, but the term structure convergence effect of single commodity pairs trading can produce the profit from pairs trading in energy markets. This result supports that the source of profits from energy futures pairs trading lies in the term structure convergence of a single commodity.

In order to obtain a deeper understanding of the yield curve play, we examined the volatility of three energy futures price returns illustrated in Figure 12. The figure shows that volatility

[18] Each commodity pairs trading introduces the selection of 15 pairs. Since cross commodity trading uses all three energy futures, it selects 45 pairs (three times as many as the selection for single commodity trades).

decreases in the maturity month, resulting in the Samuelson effect. According to Hong (2000), his model predicts that the Samuelson effect holds in markets where information asymmetry among investors is small. Thus, there may exist small information asymmetry among investors in the three energy futures markets. In the presence of small information asymmetry, investors recognize that the temporal supply shock observed in short maturity futures is just transient and dies out quickly. Thus, long maturity futures are not affected by this temporal shock, meaning that the price spread is only determined by the short-term futures price because the long-term futures price almost remains constant. When the shock settles, the price spread may converge. This result also supports our finding that the source of profits from energy futures pairs trading lies in the term structure convergence of the energy futures market we study.

[INSERT FIGURE 12 ABOUT HERE]

In addition, Litzenberger and Rabinowitz (1995) showed that strong backwardation is positively correlated with the riskiness of futures prices. If this riskiness is captured by volatility, backwardation may also be considered a source of profits from pairs trading in energy futures markets.

5. Conclusions

In this paper we examine the usefulness of a hedge fund trading strategy known as “pairs trading” as applied to energy futures markets, focusing on the characteristics of energy futures. The profit of a simplified pairs trading strategy was modeled by using a mean-reverting process of energy futures price spreads. The comparative statics of the expected return using the model indicated that both strong mean reversion and high volatility of price spreads give rise to high expected returns from pairs trading. Our empirical analyses using WTI crude oil, heating oil, and natural gas futures traded on the NYMEX show that pairs trading in energy futures markets can produce a relatively stable profit.

The sources of the total profit were investigated from the characteristics of energy futures prices: strong seasonality, strong mean reversion, high volatility, and large price spikes. The total profits from heating oil and natural gas trading were found to be positively affected by seasonality, contrary to the WTI crude oil, resulting in the greater total profits of heating oil and natural gas with seasonality than that of WTI crude oil without seasonality. Seasonality may seem to characterize the total profit. Then, we examined the influence of mean reversion and volatility of price spreads on the total profit. The results suggest that the strong mean reversion and high volatility may cause high total profits from pairs trading, especially in natural gas trades. Moreover, event risk, as measured by price spikes for pairs trading, were examined using Engle's dynamic conditional correlation model. The low correlations of natural gas futures prices with different maturities were prominently observed during the first quarter of 2003 when the Iraq war broke out, contrary to the other two energy futures. The results suggest that natural gas futures trades are the most vulnerable of the three energy futures to event risk as measured by price spikes. Thus, price spikes may also lead to the strong deterioration of the total profit from pairs trading.

Finally, we investigated the profitability of cross commodities pairs trading. We found that the portfolio effect using cross commodities may not improve the profitability of pairs trading but the term structure convergence effect of single commodity pairs trading produces the total profit from pairs trading in energy markets.

Appendix. DCC Model for Price Returns

We model the log return of the prices y_t by using Engle's dynamic conditional correlation (DCC) model as follows:[19]

$$y_t = \varepsilon_t, \tag{A1}$$

$$\varepsilon_t = D_t \eta_t, \tag{A2}$$

$$D_t = \mathrm{diag}[h_{1,t}^{\frac{1}{2}} h_{2,t}^{\frac{1}{2}}], \tag{A3}$$

where $y_t = (y_{1,t}, y_{2,t})'$, $\varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{2,t})'$, and $\eta_t = (\eta_{1,t}, \eta_{2,t})'$.

For $i = 1,\ 2$, we have

$$h_{i,t} = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{i,t-1}, \tag{A4}$$

$$E[\varepsilon_t \varepsilon_t' \mid F_{t-1}] = D_t R_t D_t, \tag{A5}$$

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1}, \tag{A6}$$

$$Q_t = (1 - \theta_1 - \theta_2) Q + \theta_1 \eta_{t-1} \eta_{t-1}' + \theta_2 Q_{t-1}, \tag{A7}$$

where Q_t^* is the diagonal component of the square root of the diagonal elements of Q_t. Equation (A4) represents a GARCH(1,1) effect for each price return. The conditional correlation is calculated as equation (A6) where equation (A7) represents the time-varying conditional covariance. If either of the estimates of θ_1 or θ_2 as in equation (A7) is statistically significant, the correlation structure of the pairs becomes time varying.

We estimated the parameters of the DCC model for the price returns and then calculated the conditional correlation by employing 1- and 2-month maturity futures prices of WTI, HO, and NG, respectively. The estimates of the parameters and the conditional correlation for WTI, HO, and NG are given in Table 13 and Figure 13, Table 14 and Figure 14, and Table 15 and Figure 15, respectively. Judging from Figures 13, 14, and 15, the correlation of WTI futures is high and stable, that of HO futures is volatile, and that of NG futures is high and stable except for the first quarter of 2003.

[INSERT TABLE 13 ABOUT HERE]

[INSERT FIGURE 13 ABOUT HERE]

[INSERT TABLE 14 ABOUT HERE]

[19] We estimated the AR(1) model for y_t, but both the AR(1) coefficient and the constant terms were not statistically significant. Thus, we model returns by equation (A1).

[INSERT FIGURE 14 ABOUT HERE]

[INSERT TABLE 15 ABOUT HERE]

[INSERT FIGURE 15 ABOUT HERE]

We next shed light on event risk where the Iraq war broke out during the first three months of 2003. Figure 13 for WTI suggests that event risk does not seem to influence the correlation of WTI futures prices. Although Figure 14 for HO suggests that event risk may more or less influence the correlation, we cannot distinguish this influence from the volatile correlation during the entire observation period. However, Figure 15 for NG suggests that event risk strongly affects that of the NG trades. Thus, the analyses employing the DCC model suggest that NG futures trades are the most vulnerable of the three energy futures to event risk, agreeing with the results reported in Section 4.3.

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Figures & Tables

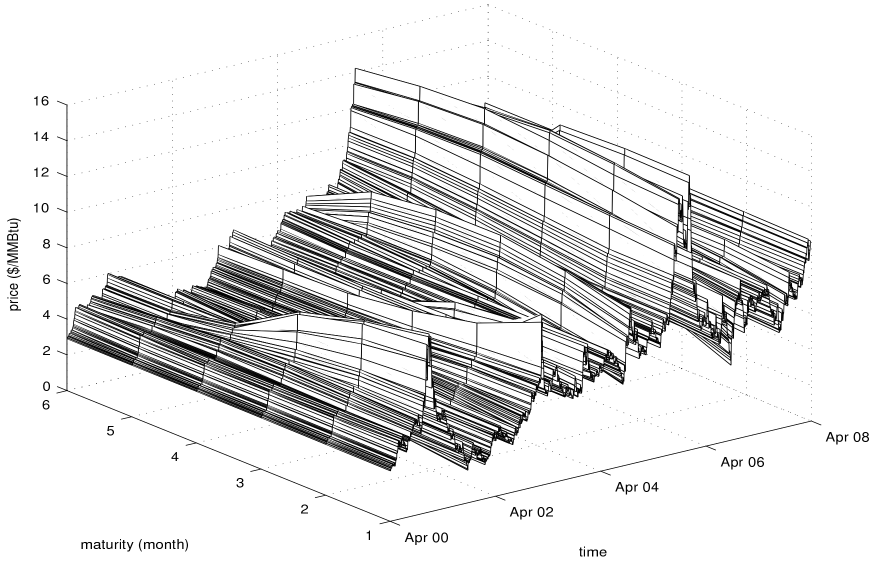

Figure 1. NYMEX Natural Gas Futures Curves

April 3, 2000 - March 31, 2008. NG futures product includes six delivery months – from one to six months.

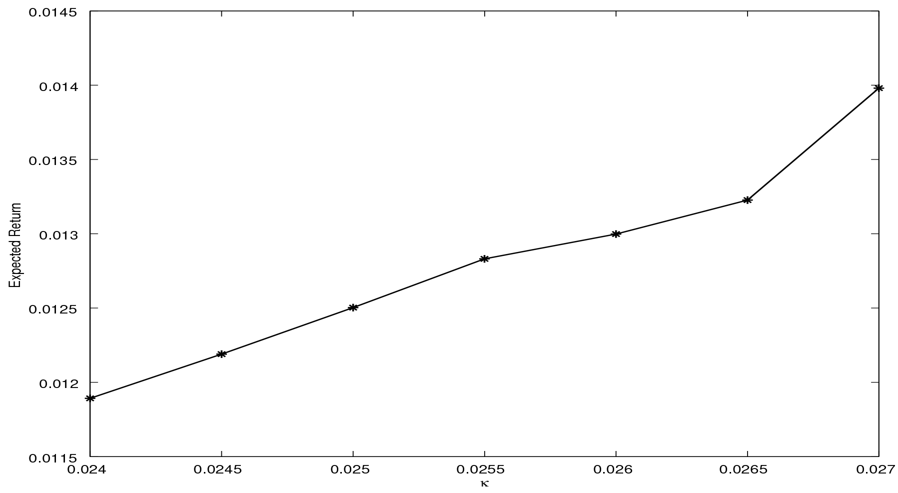

Figure 2. Comparative Statics of κ

By increasing the mean-reversion κ from 0.024 to 0.027 holding volatility constant at 0.0130, we calculated the corresponding expected returns for the simple profit model.

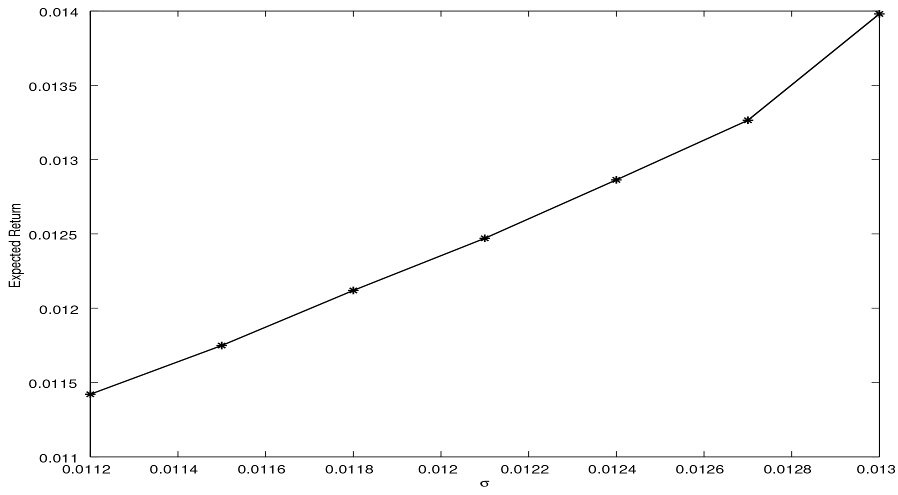

Figure 3. Comparative Statics of σ

By increasing the volatility σ from 0.0112 to 0.0130 holding mean-reversion constant at 0.027, the corresponding expected returns for the simple profit model are shown.

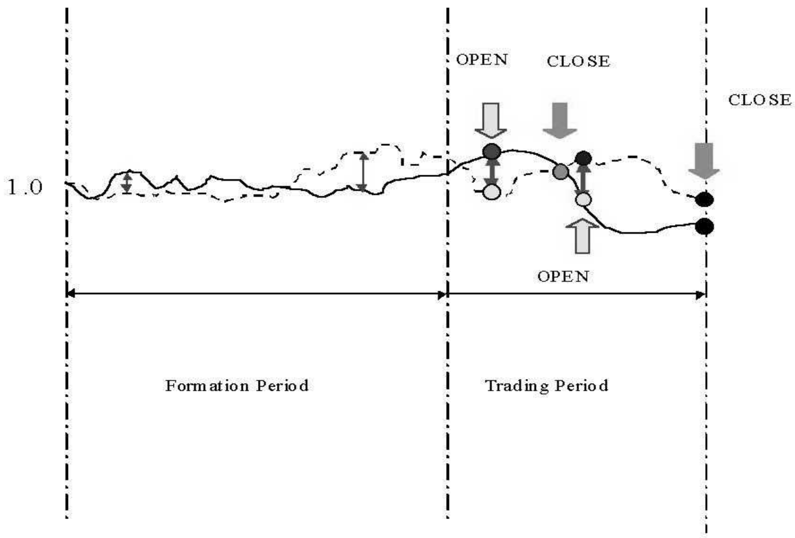

Figure 4. Pairs Trading

The prices are first normalized with the first day's price as in the figure. A pairs trading strategy involves selecting the most correlated pair of prices during the period. This period is referred to as the formation period. Then, the trade is implemented by using the selected pairs during the consecutive period, referred to as the trading period. When the price spread of the selected pairs reaches a user specified multiple of the standard deviation calculated in the formation period, a zero-cost portfolio is constructed. If the pairs converge during the trading period, then the positions are closed. Otherwise, the positions are forced to close.

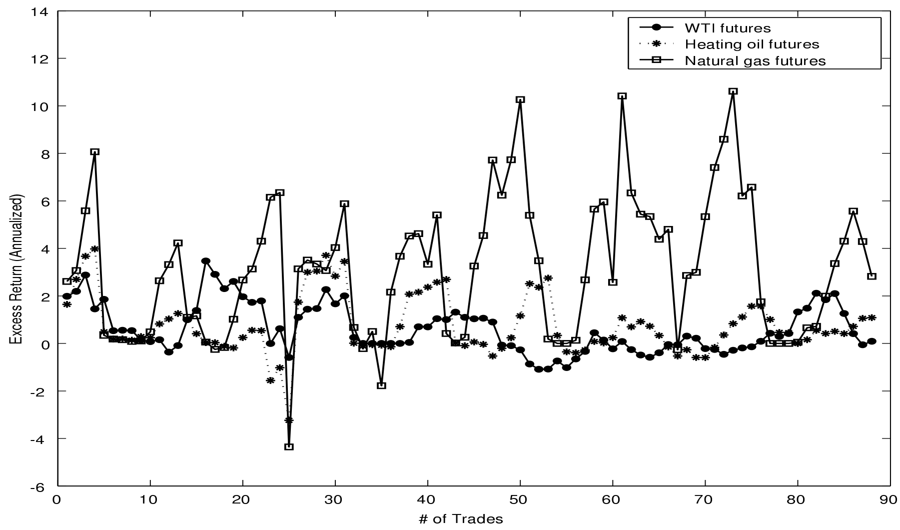

Figure 5. Pairs Trading of WTI Crude Oil, Heating Oil, and Natural Gas Futures

We set 88 formation and trading periods whose starting points are determined by rolling over 20 trading days. The first starting point is September 25, 2000 and the last (88th) point is September 14, 2007.

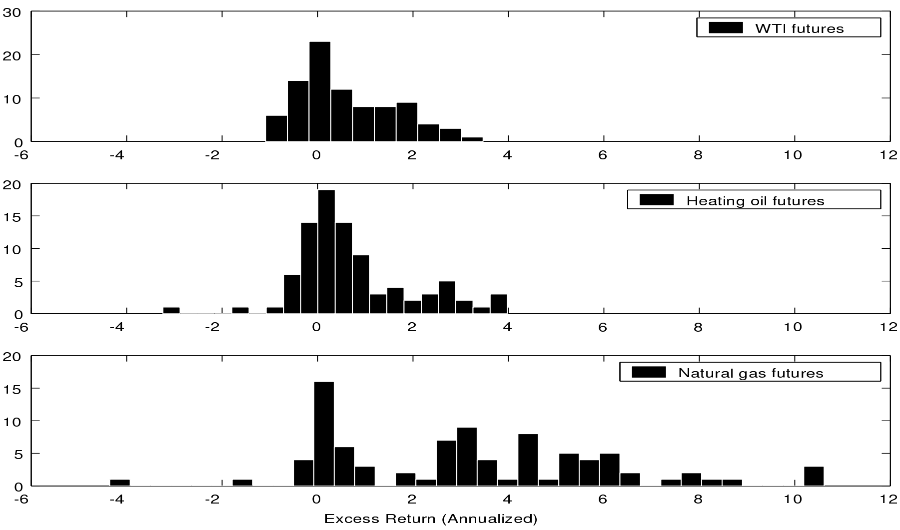

Figure 6. Returns Distributions of Pairs Trading for Three Energy Futures

(1) For returns distributions from WTI futures trades, the mean is 0.61, the maximum is 3.47, the minimum is -1.09, the standard deviation is 1.01, the skewness is 0.63, and the kurtosis is 2.68.
(2) For returns distributions from HO futures trades, the mean is 0.74, the maximum is 3.98, the minimum is -3.24, the standard deviation is 1.24, the skewness is 0.52, and the kurtosis is 3.94.
(3) For returns distributions from NG futures trades, the mean is 3.15, the maximum is 10.61, the minimum is -4.35, the standard deviation is 2.92, the skewness is 0.36, and the kurtosis is 2.90.

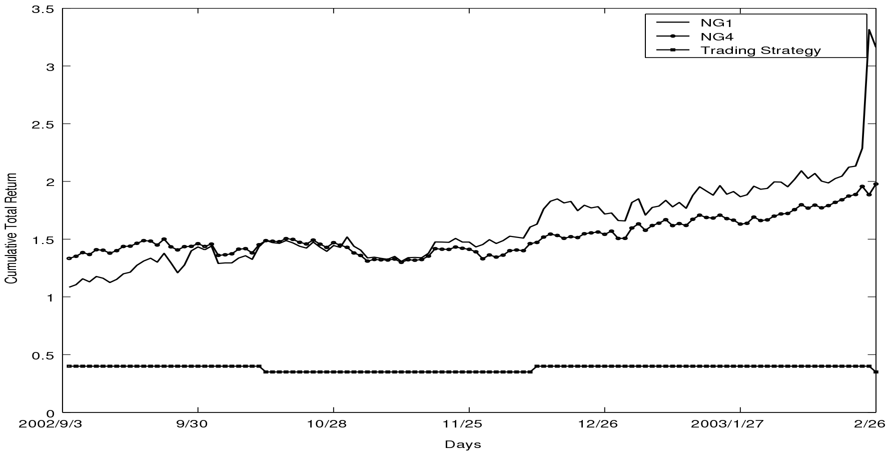

Figure 7. Pairs Trading with Event Risk

April 3, 2000 - July 10, 2003. The total returns of NG 1- and 4-month futures are shown by lines without markers and with circle markers, respectively. The total return for the trading strategy is shown by a line with square markers. If the trade is open, the line with square markers is described as the upper stair and otherwise, it is as the lower stair.

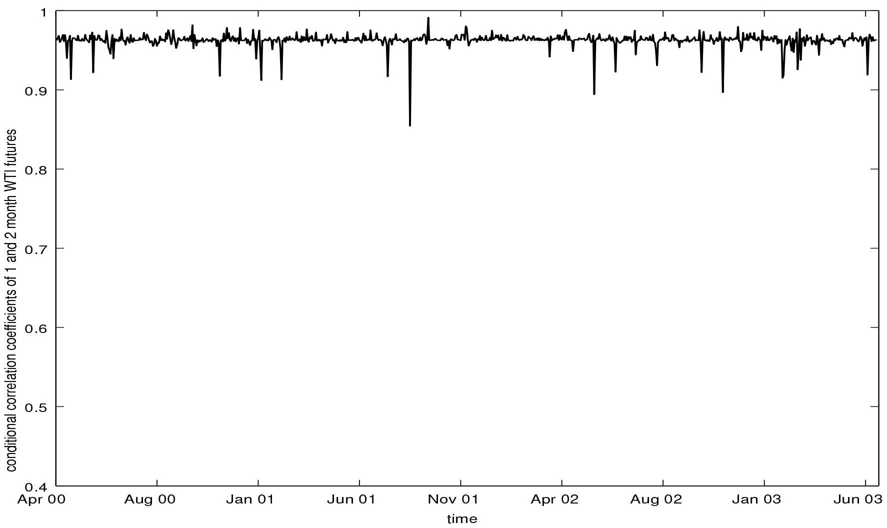

Figure 8. Dynamic Conditional Correlations of 1- and 2-Month WTI Futures Prices

April 3, 2000 - July 10, 2003. The Iraq war broke out at the end of February 2003.

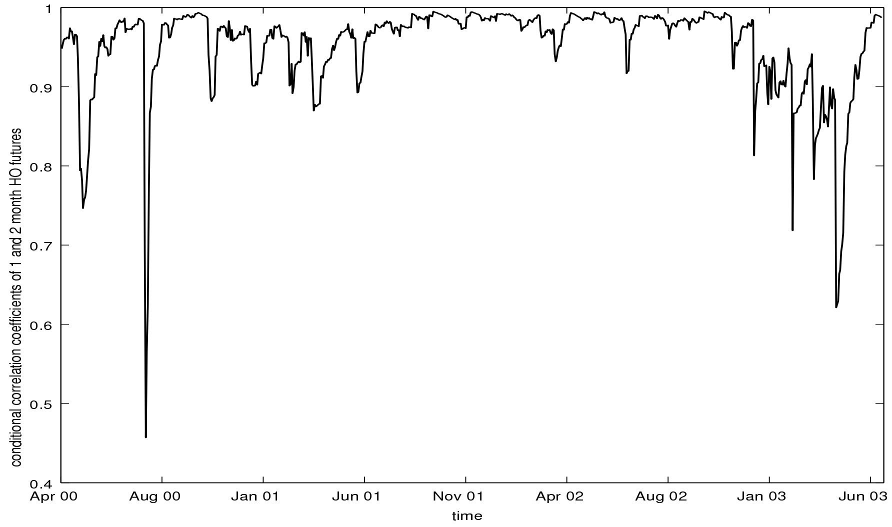

Figure 9. Dynamic Conditional Correlations of 1- and 2-Month HO Futures Prices
April 3, 2000 - July 10, 2003. The Iraq war broke out at the end of February 2003.

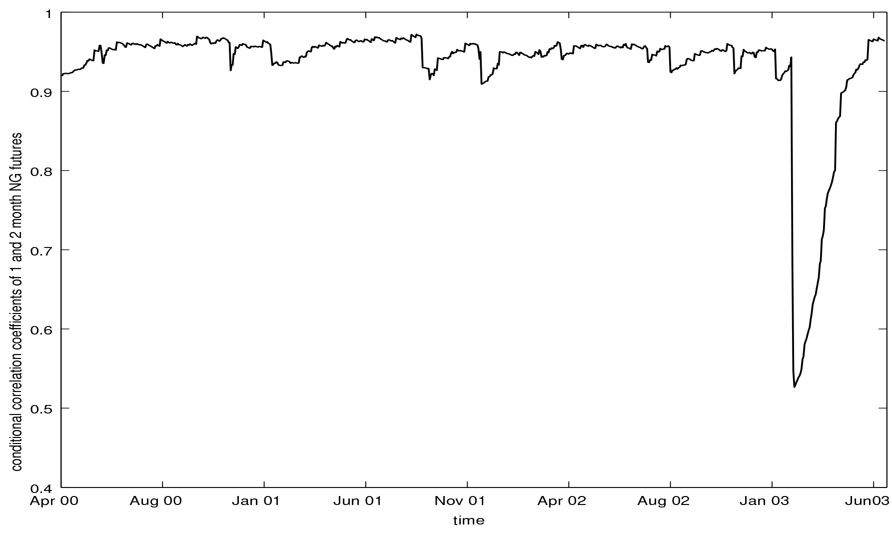

Figure 10. Dynamic Conditional Correlations of 1- and 2-Month NG Futures Prices
April 3, 2000 - July 10, 2003. The Iraq war broke out at the end of February 2003.

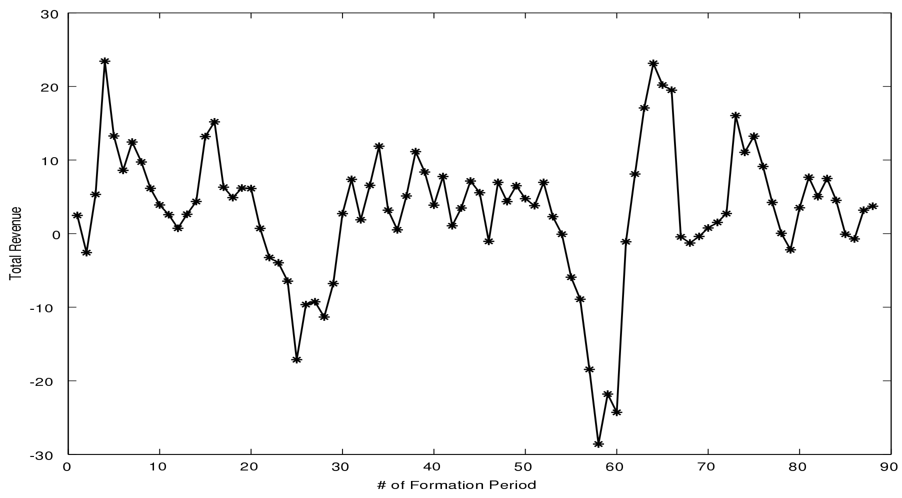

Figure 11. Returns of Pairs Trading for Cross Commodities

There are 88 formation and trading periods whose starting points are determined by rolling over 20 trading days. The first starting point is September 25, 2000 and the last (88th) is September 14, 2007.

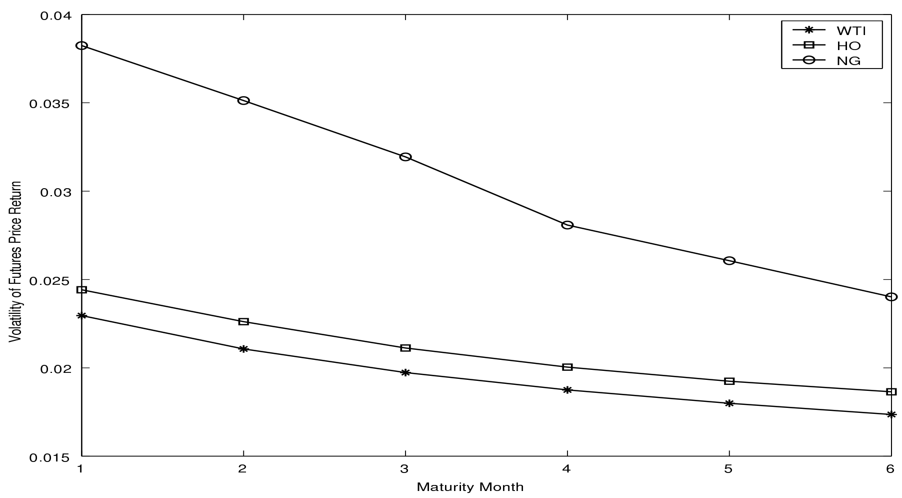

Figure 12. Samuelson Effect in Energy Futures Markets

April 3, 2000 - March 31, 2008. The volatility of price returns for WTI, HO, and NG futures – from one to six months, respectively, are shown.

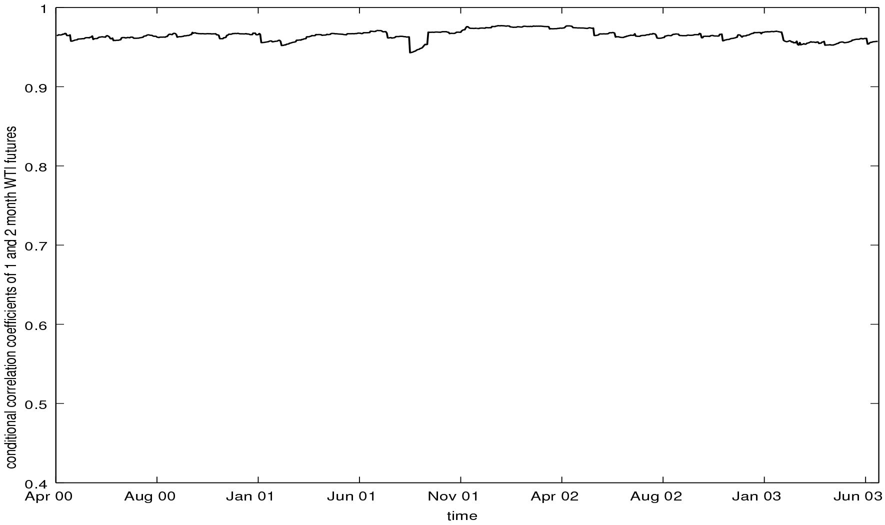

Figure 13. Dynamic Conditional Correlations of 1- and 2-Month WTI Futures Price Returns
April 3, 2000 - July 10, 2003. The Iraq war broke out at the end of February 2003.

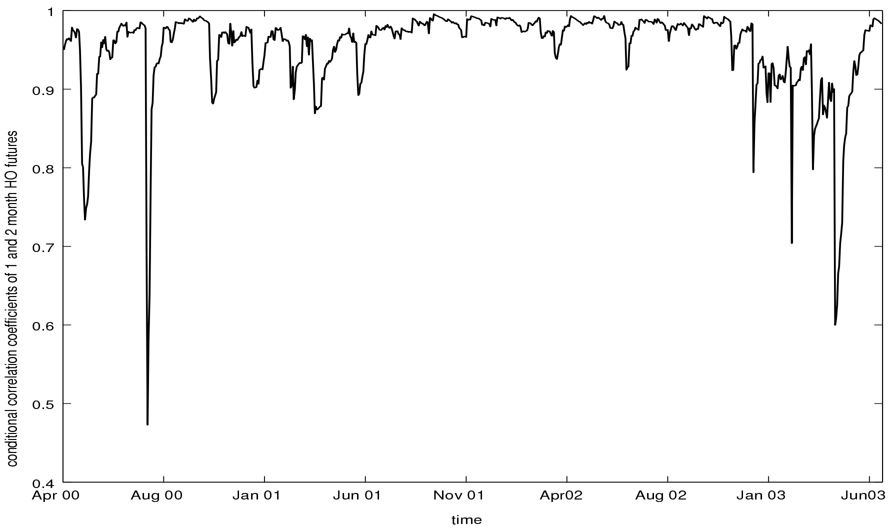

Figure 14. Dynamic Conditional Correlations of 1- and 2-Month HO Futures Price Returns
April 3, 2000 - July 10, 2003. The Iraq war broke out at the end of February 2003.

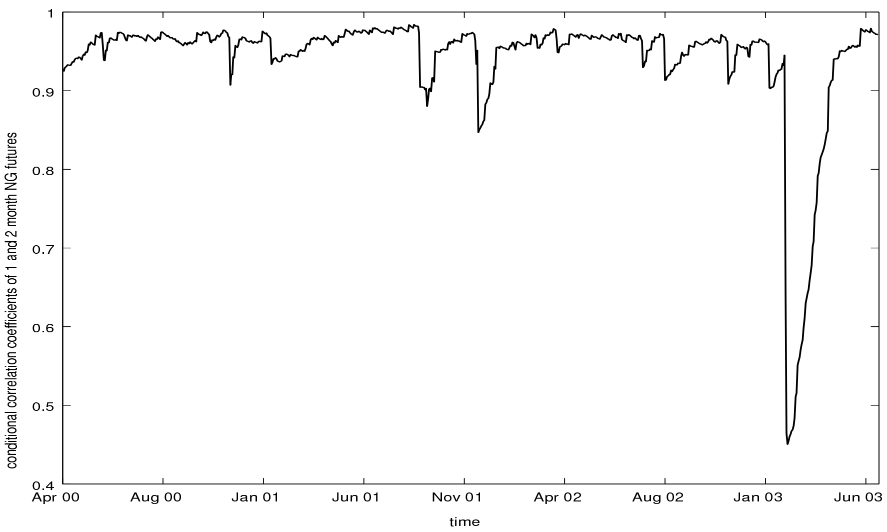

Figure 15. Dynamic Conditional Correlations of 1- and 2-Month NG Futures Price Returns
April 3, 2000 - July 10, 2003. The Iraq war broke out at the end of February 2003.

Variable	C_{12}	ρ_{12}	C_{13}	ρ_{13}	C_{14}	ρ_{14}	C_{15}	ρ_{15}	C_{16}	ρ_{16}
Coefficient	-0.152	0.957	-0.259	0.977	-0.303	0.985	-0.344	0.987	-0.363	0.989
Std. Error	0.047	0.015	0.132	0.011	0.259	0.008	0.320	0.007	0.385	0.006
Variable	C_{23}	ρ_{23}	C_{24}	ρ_{24}	C_{25}	ρ_{25}	C_{26}	ρ_{26}	C_{34}	ρ_{34}
Coefficient	-0.107	0.976	-0.151	0.985	-0.190	0.987	-0.210	0.989	-0.043	0.980
Std. Error	0.077	0.012	0.213	0.009	0.278	0.007	0.362	0.006	0.126	0.018
Variable	C_{35}	ρ_{35}	C_{36}	ρ_{36}	C_{45}	ρ_{45}	C_{46}	ρ_{46}	C_{56}	ρ_{56}
Coefficient	-0.082	0.984	-0.101	0.987	-0.039	0.976	-0.057	0.986	-0.017	0.973
Std. Error	0.197	0.011	0.296	0.007	0.077	0.012	0.195	0.007	0.089	0.018

Table 1. AR(1) Models for Natural Gas Futures Price Spread

C_{ij} and ρ_{ij} denote the constant term and the first lag coefficient of AR(1) model for price spreads ($P^i - P^j$) for i and j month natural gas futures ($i \neq j$).

	WTI_1	WTI_2	WTI_3	WTI_4	WTI_5	WTI_6
Mean	45.96	46.03	45.99	45.87	45.71	45.55
Median	37.21	36.47	35.91	35.44	34.99	34.53
Maximum	110.33	109.17	107.94	106.90	106.06	105.44
Minimum	17.45	17.84	18.06	18.27	18.44	18.60
Std. Dev.	20.66	20.92	21.14	21.32	21.49	21.65
Skewness	0.81	0.76	0.72	0.69	0.67	0.66
Kurtosis	2.79	2.59	2.44	2.33	2.24	2.17

Table 2. Basic Statistics of WTI Crude Oil Futures Prices

April 3, 2000 - March 31, 2008. The maturity month is denoted by a subindex from one to six months.

	HO_1	HO_2	HO_3	HO_4	HO_5	HO_6
Mean	127.86	128.27	128.35	128.15	127.82	127.44
Median	101.90	99.83	98.53	96.25	94.02	91.97
Maximum	314.83	306.45	301.55	301.05	301.10	301.50
Minimum	49.99	51.31	51.71	51.96	51.52	50.87
Std. Dev.	59.54	60.28	60.98	61.52	61.93	62.30
Skewness	0.73	0.68	0.65	0.64	0.63	0.62
Kurtosis	2.58	2.37	2.21	2.11	2.04	2.00

Table 3. Basic Statistics of Heating Oil Futures Prices

April 3, 2000 - March 31, 2008. The maturity month is denoted by a subindex from one to six months.

	NG_1	NG_2	NG_3	NG_4	NG_5	NG_6
Mean	6.01	6.16	6.27	6.31	6.35	6.36
Median	5.94	6.11	6.19	6.09	6.11	6.18
Maximum	15.38	15.43	15.29	14.91	14.67	14.22
Minimum	1.83	1.98	2.08	2.18	2.26	2.33
Std. Dev.	2.27	2.32	2.36	2.34	2.33	2.30
Skewness	0.90	0.90	0.90	0.73	0.60	0.39
Kurtosis	4.75	4.72	4.57	3.83	3.28	2.43

Table 4. Basic Statistics of Natural Gas Futures Prices

April 3, 2000 - March 31, 2008. The maturity month is denoted by a subindex from one to six months.

#	Beginning	End	WTI	HO	NG	Winter Trade
1	09/25/00	03/19/01	1.98	1.64	2.61	Yes
2	10/23/00	04/17/01	2.19	2.70	3.07	Yes
3	11/20/00	05/15/01	2.88	3.67	5.58	Yes
4	12/20/00	06/13/01	1.45	3.98	8.06	Yes
5	01/22/01	07/12/01	1.85	0.49	0.34	No
6	02/20/01	08/09/01	0.53	0.19	0.18	No
7	03/20/01	09/07/01	0.55	0.19	0.16	No
8	04/18/01	10/11/01	0.54	0.14	0.08	No
9	05/16/01	11/08/01	0.15	0.29	0.11	No
10	06/14/01	12/10/01	0.08	0.24	0.49	No
11	07/13/01	01/10/02	0.16	0.82	2.64	No
12	08/10/01	02/08/02	-0.36	1.03	3.32	Yes
13	09/10/01	03/11/02	-0.09	1.26	4.23	Yes
14	10/12/01	04/09/02	1.00	1.01	1.10	Yes
15	11/09/01	05/07/02	1.39	0.41	1.16	Yes
16	12/11/01	06/05/02	3.47	0.03	0.06	Yes
17	01/11/02	07/03/02	2.90	0.03	-0.25	No
18	02/11/02	08/02/02	2.30	-0.12	-0.17	No
19	03/12/02	08/30/02	2.61	-0.19	1.02	No
20	04/10/02	09/30/02	1.96	0.25	2.67	No
21	05/08/02	10/28/02	1.73	0.56	3.14	No
22	06/06/02	11/25/02	1.79	0.54	4.30	No
23	07/08/02	12/26/02	-0.01	-1.56	6.15	No
24	08/05/02	01/27/03	0.62	-1.02	6.35	No
25	09/03/02	02/25/03	-0.59	-3.24	-4.35	Yes
26	10/01/02	03/25/03	1.10	1.74	3.14	Yes
27	10/29/02	04/23/03	1.44	3.00	3.51	Yes
28	11/26/02	05/21/03	1.47	3.04	3.34	Yes
29	12/27/02	06/19/03	2.27	3.71	3.07	Yes
30	01/28/03	07/18/03	1.67	2.83	4.03	No
31	02/26/03	08/15/03	2.00	3.45	5.88	No
32	03/26/03	09/15/03	0.25	0.01	0.67	No
33	04/24/03	10/13/03	0.00	-0.11	-0.22	No
34	05/22/03	11/10/03	0.00	-0.06	0.50	No
35	06/20/03	12/10/03	0.00	-0.08	-1.78	No
36	07/21/03	01/13/04	0.00	-0.14	2.16	No
37	08/18/03	02/11/04	0.00	0.71	3.67	Yes
38	09/16/03	03/11/04	0.04	2.08	4.52	Yes
39	10/14/03	04/08/04	0.70	2.16	4.62	Yes
40	11/11/03	05/07/04	0.70	2.37	3.33	Yes
41	12/11/03	06/07/04	1.04	2.57	5.40	Yes
42	01/14/04	07/07/04	1.01	2.69	0.41	No
43	02/13/04	08/04/04	1.32	0.02	0.02	No
44	03/12/04	09/01/04	1.10	-0.10	0.26	No
45	04/12/04	09/30/04	1.04	0.06	3.26	No

Table 5. Excess Returns from Pairs Trading and Seasonality

"Winter trade" is defined to be trades including the days from December to February. The winter trades in this table comprise four periods: the trades from #1 to #4, from #12 to #16, from #25 to #29, and from #37 to #41.

#	Beginning	End	WTI	HO	NG	Winter Trade
46	05/10/04	10/28/04	1.06	-0.04	4.54	No
47	06/08/04	11/29/04	0.90	-0.53	7.72	No
48	07/08/04	12/28/04	-0.07	-0.21	6.24	No
49	08/05/04	01/27/05	-0.09	0.24	7.73	No
50	09/02/04	02/25/05	-0.27	1.16	10.26	Yes
51	10/01/04	03/28/05	-0.87	2.51	5.39	Yes
52	10/29/04	04/25/05	-1.09	2.36	3.48	Yes
53	11/30/04	05/23/05	-1.08	2.75	0.19	Yes
54	12/29/04	06/21/05	-0.74	0.34	0.00	Yes
55	01/28/05	07/20/05	-1.02	-0.36	0.00	No
56	02/28/05	08/17/05	-0.65	-0.39	0.13	No
57	03/29/05	09/15/05	-0.33	-0.28	2.68	No
58	04/26/05	10/13/05	0.45	0.08	5.65	No
59	05/24/05	11/10/05	0.14	0.02	5.96	No
60	06/22/05	12/12/05	-0.23	0.24	2.57	No
61	07/21/05	01/11/06	0.08	1.08	10.41	No
62	08/18/05	02/09/06	-0.26	0.70	6.33	Yes
63	09/16/05	03/10/06	-0.49	0.92	5.44	Yes
64	10/14/05	04/07/06	-0.58	0.73	5.34	Yes
65	11/11/05	05/08/06	-0.39	0.33	4.38	Yes
66	12/13/05	06/06/06	-0.05	-0.15	4.80	Yes
67	01/12/06	07/05/06	-0.06	-0.53	-0.26	No
68	02/10/06	08/02/06	0.31	-0.26	2.86	No
69	03/13/06	08/30/06	0.22	-0.59	2.99	No
70	04/10/06	09/28/06	-0.22	-0.60	5.34	No
71	05/09/06	10/26/06	-0.26	-0.17	7.40	No
72	06/07/06	11/24/06	-0.46	0.36	8.59	No
73	07/06/06	12/22/06	-0.29	0.83	10.61	No
74	08/03/06	01/25/07	-0.19	1.11	6.20	No
75	08/31/06	02/23/07	-0.14	1.57	6.58	Yes
76	09/29/06	03/23/07	0.09	1.58	1.75	Yes
77	10/27/06	04/23/07	0.43	1.01	0.00	Yes
78	11/27/06	05/21/07	0.30	0.47	0.00	Yes
79	12/26/06	06/19/07	0.42	0.45	0.00	Yes
80	01/26/07	07/18/07	1.33	0.00	0.05	No
81	02/26/07	08/15/07	1.48	0.16	0.66	No
82	03/26/07	09/13/07	2.11	0.55	0.72	No
83	04/24/07	10/11/07	1.84	0.42	1.99	No
84	05/22/07	11/09/07	2.10	0.51	3.36	No
85	06/20/07	12/07/07	1.26	0.41	4.31	No
86	07/19/07	01/09/07	0.41	0.72	5.56	No
87	08/16/07	02/06/07	-0.05	1.05	4.29	Yes
88	09/14/07	03/06/07	0.09	1.08	2.82	Yes
Average Excess Return			0.61	0.74	3.15	
Variance of the Return			1.03	1.53	8.52	

Table 6. Excess Returns from Pairs Trading and Seasonality (Cont'd)

"Winter trade" is defined to be trades including the days from December to February. The winter trades in this table comprise four periods: from #50 to #54, from #62 to #66, from #75 to #79, and from #87 to #88.

	AR(1) Coefficient ($\hat{\rho}$)	Standard Deviation ($\hat{\sigma}$)	Average Excess Return
WTI Futures Spread	0.994	0.062	0.61
Heating Oil Futures Spread	0.990	0.083	0.74
Natural Gas Futures Spread	0.981	0.257	3.15

Table 7. Mean-Reversion and Volatility

For all combinations of the spreads for each energy futures, the parameters of AR(1) coefficient ρ and standard deviation σ are estimated. From these estimates the average $\bar{\rho}$ and $\bar{\sigma}$ are computed. The average excess return is obtained from Table 6.

	α	β	μ	σ
WTI	2.000	1.000	0.302	0.758
HO	1.464	1.000	0.940	0.539
NG	2.000	1.000	3.101	2.613

Table 8. Parameter Estimation of α-stable distribution

α-stable distribution is often introduced as a tool to model high skewness and kurtosis. Unfortunately, it does not have distribution function and density that is in closed form. Stable distributions are introduced by their characteristic function as follows,

$$\log F(t) = \begin{cases} -\sigma^{\alpha} \mid t \mid^{\alpha} \left(1 - i\beta \operatorname{sgn}(t) \tan(\frac{\pi\alpha}{2})\right) + i\mu t, & \alpha \neq 1 \\ -\sigma \mid t \mid \left(1 + i\beta \frac{2}{\pi} \operatorname{sgn}(t) \log \mid t \mid \right) + i\mu t, & \alpha = 1, \end{cases}$$

where $F(t)$ denotes the characteristic function of the stable law:

$$F(t) = \int e^{itx} \frac{1}{\sigma} f\left(\frac{x-\mu}{\sigma}\right) dx.$$

		$\phi_{1,0}$	$\phi_{1,1}$	$\phi_{2,0}$	$\phi_{2,1}$
WTI	Coefficient	28.307	0.984	27.995	0.985
	Std. Error	1.641	0.007	1.604	0.007
HO	Coefficient	77.245	0.985	76.838	0.987
	Std. Error	5.246	0.008	5.235	0.006
NG	Coefficient	4.547	0.988	4.641	0.991
	Std. Error	0.741	0.009	0.824	0.007

Table 9. AR(1) Model for Futures Prices

The parameter estimation results of the AR(1) models for WTI, HO, and NG futures prices are reported where $\phi_{1,0}$ and $\phi_{2,0}$ represent the constant terms for 1- and 2-month maturity futures prices, respectively, and $\phi_{1,1}$ and $\phi_{2,1}$ represent the corresponding AR(1) coefficients, respectively.

Parameters	ω_1	α_1	β_1	ω_2	α_2	β_2	θ_1	θ_2
Estimates	2.309×10^{-2}	0.080	0.879	3.783×10^{-2}	0.100	0.808	0.093	0.000
Std Errors	1.892×10^{-2}	0.047	0.070	1.626×10^{-2}	0.036	0.056	0.064	4.069
Loglikelihood	-5.672×10^{2}							
AIC	1.150×10^{3}							
SIC	1.188×10^{3}							

Table 10. Parameter Estimates of DCC Model for WTI Futures Prices

ARCH and GARCH terms are denoted by α_i and β_i, respectively, where the subindex i represents i-month futures. If the estimates of α_i and β_i are statistically significant, there exists a GARCH effect in i-month WTI futures price noises. The covariance matrix is represented by the past noise and the past covariance matrix whose coefficients are represented by θ_1 and θ_2, respectively. If either of the estimates of θ_1 or θ_2 is statistically significant, the correlation of the pairs of WTI futures prices becomes time varying.

Parameters	ω_1	α_1	β_1	ω_2	α_2	β_2	θ_1	θ_2
Estimates	2.203×10^{-1}	0.145	0.813	1.312×10^{-1}	0.099	0.868	0.123	0.859
Std Errors	8.923×10^{-2}	0.047	0.048	6.819×10^{-2}	0.039	0.042	0.057	0.068
Loglikelihood	-2.240×10^{3}							
AIC	4.496×10^{3}							
SIC	4.534×10^{3}							

Table 11. Parameter Estimates of DCC Model for HO Futures Prices

ARCH and GARCH terms are denoted by α_i and β_i, respectively, where the subindex i represents i-month futures. If the estimates of α_i and β_i are statistically significant, there exists a GARCH effect in i-month HO futures price noises. The covariance matrix is represented by the past noise and the past covariance matrix whose coefficients are represented by θ_1 and θ_2, respectively. If either of the estimates of θ_1 or θ_2 is statistically significant, the correlation of the pairs of HO futures prices becomes time varying.

Parameters	ω_1	α_1	β_1	ω_2	α_2	β_2	θ_1	θ_2
Estimates	2.956×10^{-3}	0.365	0.635	1.231×10^{-3}	0.150	0.812	0.036	0.944
Std Errors	1.400×10^{-3}	0.213	0.132	4.356×10^{-4}	0.036	0.035	0.070	0.016
Loglikelihood	1.448×10^{3}							
AIC	-2.881×10^{3}							
SIC	-2.843×10^{3}							

Table 12. Parameter Estimates of DCC Model for NG Futures Prices

ARCH and GARCH terms are denoted by α_i and β_i, respectively, where the subindex *i* represents *i*-month futures. If the estimates of α_i and β_i are statistically significant, there exists a GARCH effect in *i*-month NG futures price noises. The covariance matrix is represented by the past noise and the past covariance matrix whose coefficients are represented by θ_1 and θ_2, respectively. If either of the estimates of θ_1 or θ_2 is statistically significant, the correlation of the pairs of NG futures prices becomes time varying.

Parameters	ω_1	α_1	β_1	ω_2	α_2	β_2	θ_1	θ_2
Estimates	5.187×10^{-5}	0.087	0.841	0.000	0.104	0.807	0.011	0.974
Std Errors	2.672×10^{-5}	0.042	0.060	0.000	0.040	0.054	0.064	0.205
Loglikelihood	4.818×10^{3}							
AIC	-9.621×10^{3}							
SIC	-9.583×10^{3}							

Table 13. Parameter Estimates of DCC Model for WTI Futures Price Returns

ARCH and GARCH terms are denoted by α_i and β_i, respectively, where the subindex *i* represents *i*-month futures. If the estimates of α_i and β_i are statistically significant, there exists a GARCH effect in *i*-month WTI futures price return noises. The covariance matrix is represented by the past noise and the past covariance matrix whose coefficients are represented by θ_1 and θ_2, respectively. If either of the estimates of θ_1 or θ_2 is statistically significant, the correlation of the pairs of WTI futures price returns becomes time varying.

Parameters	ω_1	α_1	β_1	ω_2	α_2	β_2	θ_1	θ_2
Estimates	5.490×10^{-5}	0.123	0.809	0.000	0.066	0.883	0.129	0.846
Std Errors	2.386×10^{-5}	0.053	0.063	0.000	0.043	0.076	0.060	0.083
Loglikelihood	4.764×10^{3}							
AIC	-9.512×10^{3}							
SIC	-9.475×10^{3}							

Table 14. Parameter Estimates of DCC Model for HO Futures Price Returns
ARCH and GARCH terms are denoted by α_i and β_i, respectively, where the subindex *i* represents *i*-month futures. If the estimates of α_i and β_i are statistically significant, there exists a GARCH effect in *i*-month HO futures price return noises. The covariance matrix is represented by the past noise and the past covariance matrix whose coefficients are represented by θ_1 and θ_2, respectively. If either of the estimates of θ_1 or θ_2 is statistically significant, the correlation of the pairs of HO futures price returns becomes time varying.

Parameters	ω_1	α_1	β_1	ω_2	α_2	β_2	θ_1	θ_2
Estimates	2.031×10^{-4}	0.168	0.739	0.000	0.084	0.863	0.060	0.917
Std Errors	1.453×10^{-4}	0.123	0.150	0.000	0.032	0.040	0.049	0.033
Loglikelihood	3.840×10^{3}							
AIC	-7.664×10^{3}							
SIC	-7.627×10^{3}							

Table 15. Parameter Estimates of DCC Model for NG Futures Price Returns
ARCH and GARCH terms are denoted by α_i and β_i, respectively, where the subindex *i* represents *i*-month futures. If the estimates of α_i and β_i are statistically significant, there exists a GARCH effect in *i*-month NG futures price return noises. The covariance matrix is represented by the past noise and the past covariance matrix whose coefficients are represented by θ_1 and θ_2, respectively. If either of the estimates of θ_1 or θ_2 is statistically significant, the correlation of the pairs of NG futures price returns becomes time varying.