Implied Correlations in CDO Tranches

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Abstract

Market quotes of CDO tranches constitute a market view on correlation at different points in the portfolio capital structure and thus on the shape of the portfolio loss distribution. We investigate different calibrations of the CreditRisk+ model to examine its ability to reproduce iTraxx tranche quotes. Using initial model calibration, CreditRisk+ clearly underestimates senior tranche losses. While sensitivities to correlation are too low, by increasing PD volatility up to about 3 times of the default probability for each name CreditRisk+ produces tails which are fat enough to meet market tranche losses. Additionally, we find that, similar to the correlation skew in the large pool model, to meet market quotes for each tranche a different PD volatility vector has to be used.

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1 Introduction

Credit derivatives started actively trading in the mid 1990s, exhibiting impressive growth rates over the last years. Due to new regulatory requirements there is an increasing demand by holders of securitisable assets to sell assets or to transfer risks of their assets. Other market participants seek for arbitrage opportunities which arise when derivative products are not traded at their fair price in the market. While in the beginning products were purely OTC traded, now more and more of them are liquidly traded. Not only liquidity and transparency but also the variety of products is growing. Products range from simple credit default or basket default swaps to derivatives on derivative products.

One of the most popular credit derivative products are CDOs. CDOs belong to the class of asset backed securities and typically securitise a portfolio of bonds or loans. The portfolio is split into packages which have different priorities in their claims to the collateral. By selling these securitised assets, the issuer transfers the respective part of credit risk of the collateral pool to the investor. Therefore, CDOs constitute a new means to repackage risk profiles and to meet investors individual risk return requirements in a very flexible way.

A recent development in the credit derivatives market is the availability of market quotes on CDS indices such as the iTraxx and the CDX, on which the market also has agreed to quote standard tranches. These innovations are of great importance to the credit risk world as with the emerge of these kind of standardized products the correlation increasingly becomes a market observable. As a consequence, the understanding of correlation develops and CDO pricing can be adapted to deal with the increasing level of sophistication in the market. We make use of this by drawing conclusions concerning the calibration of the widely used credit risk model CreditRisk+.

The remainder of the paper is organized as follows. Section 2-4 serve as theoretical introduction. Section 2 draws up some basics of credit risk models, focusing on the two models used in the analysis, the large pool model and CreditRisk+. Section 3 outlines the risk neutral valuation of CDOs. In section 4 the concept of base correlations, their benefits, drawbacks and potential alternatives are elaborated. Section 5 empirically studies the ability of CreditRisk+ to meet market quotes. First, we examine which implied correlations would have to be quoted in the market in order to fit our CreditRisk+ models quotes. We further investigate if this gives us insights to the correlation level used in CreditRisk+. Then we try to reproduce market quotes with varying correlations and volatilities of default probability. Section 6 concludes.

2 Credit Risk Models

In this section we briefly present some of the main ideas for correlation modelling in credit risk. Since in our empirical analysis we will use the CreditRisk+ and the CreditMetrics based homogeneous large pool Gaussian copula model, our focus is especially set on correlation modeling in these approaches.

2.1 On Correlation Modelling

A main input of credit risk models is the correlation structure of the portfolio. Data sources which are used to estimate the correlation structure are historical default data, credit spreads and equity correlation (McGinty and Ahluwalia, 2004a).

Historical default data is perhaps in theory the most appropriate data source. However, there are significant problems with this data source. First, joint defaults are rare events, so that in order to obtain a significant estimate a long history is required. Thus, it is questionable whether the correlation estimates produced from the data is relevant to the corporate world today. Second, this method neglects any company specifics. It is arguable, how accurate data from other companies is to the companies of which you estimate the correlation.

An advantage of credit spreads is that they can be more frequently observed. In contrast to both other data sources, credit spreads also reflect information about the market view of risk. However, they can also change due to other reasons than a change in the default probability of the obligor. For example the price can be influenced by liquidity factors for the particular issue.

The advantage of using equity correlations is that there is a good history of prices and data is of better quality than credit spread data. However, as with historical default data the question arises, how accurate equity correlation estimates are, since the connection between equity prices and credit risk is not too close.

Probably because equity prices are best available, using equity correlation is the current market standard. But as the credit derivative market is growing, CDS spreads may become more attractive to use.

2.2 *CreditMetrics*

The CreditMetrics framework established by JP Morgan (Gupton et al., 1997) is based on the Merton model of pricing corporate debt. According to Merton (1974), a firm defaults if its asset value falls below a certain threshold. Thus, the firm value is the underlying process which drives credit events. In contrast to other credit risk models, CreditMetrics can also handle migrations in the credit quality of an obligor, by employing a threshold for each possible migration from one credit rating to another. For simplicity however we only consider the states of default and of non-default. The default of an obligor i can be represented by a Bernoullian random variable x_i with mean p_i and standard deviation $\sigma_i = \sqrt{p_i(1-p_i)}$. A further key characteristic of the CreditMetrics model is, that the full loss distribution cannot be calculated analytically but it relies on Monte Carlo simulation. In order to conduct the simulation, we need to determine the following variables:

- for each obligor, its possible values at the end of the horizon are specified. In the case of no default, it equals the sum of its discounted cash flows. In the case of default it equals its recovery rate.
- using the assumption of standard normally distributed asset returns we can translate an historically estimated probability of default into a threshold of default for each obligor.
- In order to obtain the default correlation between two obligors i, j, we use the general correlation formula of two random variables:

$$\begin{aligned} corr(x_i, x_j) &= \frac{cov(x_i, x_j)}{\sqrt{var(x_i)var(x_j)}} \\ &= \frac{E(x_i x_j) - E(x_i)E(x_j)}{\sqrt{p_i(1-p_i)}\sqrt{p_j(1-p_j)}} \\ &= \frac{p_{ij} - p_i p_j}{\sqrt{p_i(1-p_i)}\sqrt{p_j(1-p_j)}} \end{aligned} \tag{2.1}$$

  Here the only variable which is not defined by the univariate distributions of the single obligors is the joint probability of default of the two obligors p_{ij}, which can be derived from the joint distribution of asset returns.

In order to determine the portfolio loss distribution, a large number of standard normally distributed asset returns for all obligors is drawn. According to their thresholds, these asset returns can be translated into the state of default or non-default at the end of the horizon from which the portfolio value is obtained.

2.2.1 Homogeneous Large Pool Gaussian Copula Model

The homogeneous large pool Gaussian copula model, or simply the large pool model, is a simplified form of the original CreditMetrics model. For the valuation of CDS and CDOs and the for estimation implied correlations it became a standard market model. It employs the following assumptions:

- The default of an obligor is triggered when its asset value falls below a certain threshold.
- The asset value is driven by one standard normally distributed factor. The factor both incorporates the market by a systematic risk component and the firm specific risk by an idiosyncratic risk component.
- The portfolio consists of a large number of credits of uniform size, uniform recovery rate and uniform probability of default

Thus, we can describe the normalized asset value of the ith obligor x_i by a one-factor model:

$$x_i = \sqrt{\varrho} m + \sqrt{1 - \varrho} z_i \tag{2.2}$$

where m denotes the normalized return of the systematic risk factor and z_i the idiosyncratic risk with $m, z_i \sim \Phi(0,1)$, and thus also $x_i \sim \Phi(0,1)$. $\sqrt{\varrho}$ is the correlation of each obligor with the market factor and ϱ the uniform pairwise correlation between the obligors. Let p denote the probability of default, thus the threshold of default equals $\Phi^{-1}(p)$. Assuming a recovery rate of 0% we can get the expected percentage portfolio loss given m

$$p(m) = \phi\left(\frac{\phi^{-1}(p) - \sqrt{\varrho} m}{\sqrt{1 - \varrho}}\right) \tag{2.3}$$

From this, an analytic expression for the portfolio loss distribution can be derived (Bluhm et al., 2003).

2.3 CreditRisk+

The CreditRisk+ model of Credit Suisse First Boston (1997) is based on a typical insurance mathematics approach as portfolio losses are modelled by means of a Poisson mixture model. Making no assumptions about the causes of default and given a large number of individual risks each with a low probability of occurring, the number of defaults is well presented by the Poisson distribution. In order to account for the phenomenon of correlated default events, obligor default rates are linked to sector variables. Let $x_k, k = 1,..,n$ be a random variable that represents the average number of default events of sector k, p_A the mean default probability, σ_A the standard deviation and $\Theta_{A,k}$ the sector weights of obligor A with $\sum_{k=1}^{n} \Theta_{Ak} = 1$ for all A.

Sector variables are assumed to be Gamma distributed with mean μ_k and standard deviation σ_k

$$\mu_k = \sum_A \Theta_{A,k} p_A \tag{2.4}$$

$$\sigma_k = \sum_A \Theta_{A,k} \sigma_A \tag{2.5}$$

Now the random default probability of obligor A x_A is modelled proportional to x_k

$$x_A = p_A \sum_{k=1}^{n} \Theta_{A,k} \frac{x_k}{\mu_k} \tag{2.6}$$

From this equation, Credit Suisse First Boston (1997) derive the conditional portfolio loss distribution and the unconditional portfolio loss distribution using convolution [1] .

2.3.1 Introducing Sector Correlation

Buergisser et al. (1999) extend the model by introducing sector correlation. Let $\gamma = (\gamma_1, ..., \gamma_n)'$ be a vector of random default variables of the sectors, normalized to the mean equal to 1, σ_k^2 the relative variance of γ_k, L_k the loss of sector k and ϵ_k the mean of L_k. Then they derive the variance of the loss distribution as:

$$var(\sum_k L_k) = \sum_A p_A \nu_A^2 + \sum_k \sigma_k^2 \varepsilon_k^2 + \sum_{k,l;k \neq l} \varepsilon_k \varepsilon_l corr(\gamma_k, \gamma_l) \sigma_k \sigma_l. \tag{2.7}$$

The first term is the risk due to the statistical nature of default events, the unsystematic risk, while the second and third expression represent the systematic part of the risk. To calculate the full loss distribution, Buergisser et al. (1999) propose to use a single factor model instead of the multi-factor model. The variance of the loss distribution using s single factor model is

[1] see Credit Suisse First Boston (1997), section A10

$$var(L_{sf}) = \sigma_{sf}^2 \varepsilon^2 + \sum_A p_A \nu_A^2 \qquad (2.8)$$

where $\varepsilon = \sum_A p_A \nu_A$. The key idea is to model the single factor such that it produces a loss distribution of which the variance is equal to the variance of the multi-factor model. Then the variance of the single factor is given by

$$\sigma_{sf}^2 = \frac{1}{\varepsilon^2} \sum_k \sigma_k^2 \varepsilon_k^2 + \frac{1}{\varepsilon^2} \sum_{k,l;k \neq l} \varepsilon_k \varepsilon_l corr(\gamma_k, \gamma_l) \sigma_k \sigma_l \qquad (2.9)$$

3 Collateralized Debt Obligations

A collateralized debt obligation (CDO) is a structured financial product that securitises a diversified pool of debt assets, called the collateral. The issuer of a CDO essentially repackages the risk profile of the collateral by splitting it into securities which have different priorities in their claims to the collateral. By selling these securitised assets, the issuer transfers the complete credit risk of the collateral pool to the investor.

By order of seniority, the tranches are called equity tranche, mezzanine tranche(s) and senior tranche (see figure 1). Consider for example a CDO which securitises a portfolio of bonds with a total notional of 100. Senior, mezzanine and equity tranche have a notional of $15, 80$, and 5, respectively. The equity tranche is the tranche which suffers first from potential losses of the underlying bond portfolio. If losses exceed the the notional of the equity tranche of 5, the mezzanine tranche is the tranche which is affected next. Finally, if the notional of both tranches, which subordinate the senior tranche, is used up, the latter suffers losses.

CDOs can roughly be classified by the following criteria [2]:

- Underlying assets: Most CDOs are based on corporate bonds *(CBOs)* and commercial loans *(CLOs)*. Further underlying assets are structured products, such as asset-backed securities and other CDOs (called *CDO squared*), and emerging market debt.
- Purpose: *Arbitrage CDOs* aim to profit from price differences between the components included in the CDO and the sale price of the CDO tranches. They securitise traded assets like bonds and credit default swaps. *Balance sheet CDOs* aim to shrink the balance sheet, reduce required economic or regulatory capital.

[2] For more detailed information about CDOs see for example Tavakoli (2003)

- Credit structure: While the portfolio of a *cash flow CDO* is not actively traded *market value CDOs* include actively traded assets where the portfolio manager has to meet fixed requirements concerning for example credit quality of the assets or diversification.
- In opposition to *cash CDOs*, *synthetic CDOs (CSOs)* are constructed using credit default swaps. There is no true sale of an underlying portfolio, but the issuer sells credit default swaps on a synthetic portfolio.

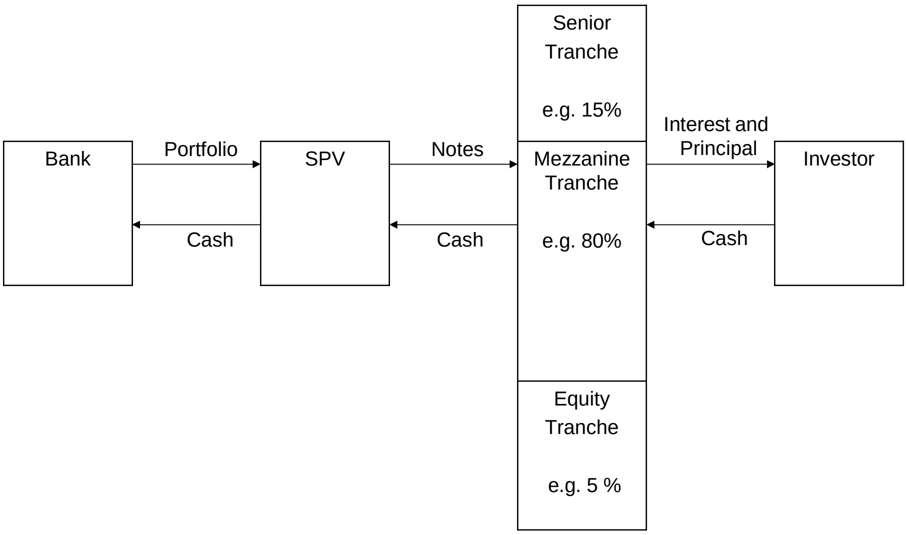

Fig. 1. Cash-flow structure of a CDO; example of bond portfolio securitisation.

3.1 Valuation of a Synthetic CDO Tranche

We assume that there exists a risk-neutral martingale measure Q, under which all price processes discounted with the interest rate process r are martingales. In this section, all expectations are with respect to this measure. The idea which underlies the valuation of synthetic CDO tranches is that the fair spread is the spread, with which the mark-to-market value of the contract is zero. Equivalently, if the issuer pays the fair spread, the present value of the fee payments is equal to the present value of the contingent payments, using risk-neutral valuation:

$$PV_{fee\ payments} = PV_{contingent\ payments} \tag{3.1}$$

First we specify how expected losses of a CDO tranche are determined by the expected portfolio loss. We therefore derive the tranche spread in terms of expected losses. We define some further notation:

K_j	upper attachment point of tranche j
S_j	spread of tranche j paid per year
$F_j(t)$	face value of tranche j at time t
$L(t)$	portfolio loss at time t
$L_j(t)$	loss of tranche j at time t
$DF(t)$	discount factor at time t
T	lifetime of the CDO, in year fraction

Let us examine the expected tranche loss at time t. A tranche only suffers losses, if the total portfolio loss excess the lower attachment point of the tranche. The maximum loss a tranche can suffer is its tranche size $K_j - K_{j-1}$. Thus, the loss of tranche j can be expressed in terms of the total portfolio loss as

$$L_j(t) = min(max(0; L(t) - K_{j-1}); K_j - K_{j-1}) \tag{3.2}$$

This is graphically illustrated in figure 2 for a mezzanine tranche with $K_{j-1} = 3\%$ and $K_j = 6\%$. Its loss profile is similar to an option spread, as a mezzanine tranche can also be interpreted as being short a call option with a strike at 3% of the total portfolio loss and being long a call option with a strike at 6% of the total portfolio loss.

The present value of the contingent payments in terms of the expected tranche loss is

$$PV_{contingent,j} = E\left(\int_0^T DF(t)dL_j(t)\right) \tag{3.3}$$

Fee payments depend on the face value at time t of the tranche $F_j(t)$, i.e. of the expected survival of the tranche, which can be written as

$$F_j(t) = (K_j - K_{j-1}) - L_j(t) \tag{3.4}$$

This gives us the expected present value of the fee payments:

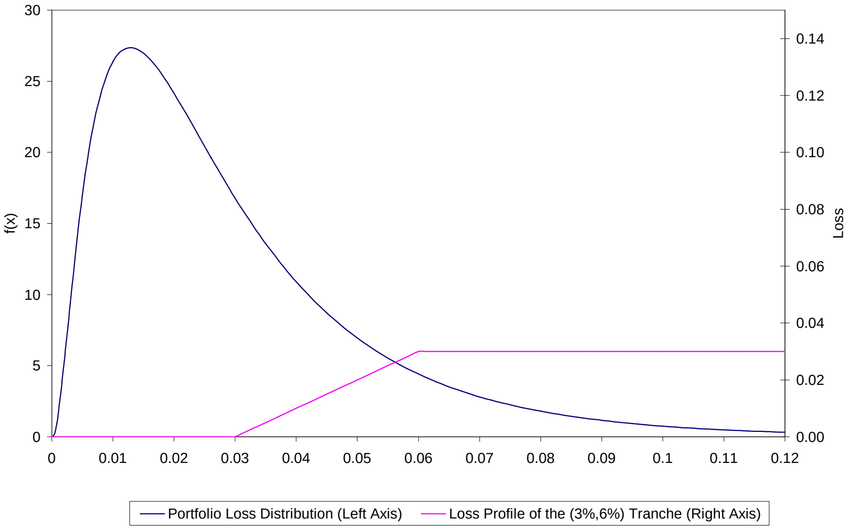

Fig. 2. Portfolio loss distribution and loss profile of a mezzanine tranche.

$$PV_{fee,j} = S_j E\left(\sum_{i=1}^{n} \delta_i DF(t_i) F_j(t_i) \right) \tag{3.5}$$

Again it can be solved for the fair spread:

$$S_{fair,\, j} = \frac{E(\int_0^T DF(t) dL_j(t))}{E(\sum_{i=1}^{n} \delta_i DF(t_i) F_j(t_i))} \tag{3.6}$$

4 Implied Correlations in CDO Tranches

4.1 The Concept of Implied Correlations

One of the latest developments in the credit derivatives market is the availability of liquidly traded standardized tranches on CDS indices. The most popular example in Europe is the iTraxx Europe. It consists of the 125 most liquidly traded European CDS which are assigned to six different industry groups. The market agreed to quoted standard tranches which are responsible for 0% to 3%, 3% to 6%, 6% to 9%, 9% to 12% and 12% to 22% of the losses. As these tranche quotes are a function of supply and demand they reflect a market view of the correlation of the underlying portfolio. From this, the concept of implied correlation or "the market view of correlation", emerged.

Mashal et al. (2004) define the implied correlation of a tranche as *the uniform asset correlation number that makes the fair or theoretical value of a tranche equal to its market quote*. In other words, Hull and White (2004) for example define the implied correlation for a tranche as *the correlation that causes the value of the tranche to be zero*.

As the concept of implied correlations is quite new, the large pool model due to its analytical tractability and small number of parameters constitutes the market standard for calculating implied correlation (McGinty and Ahluwalia, 2004a; Kakodkar et al., 2003). This enables market participants to calculate and quote implied default correlation, to trade correlation and to use implied default correlation for relative value considerations when comparing alternative investments in synthetic CDO tranches or to make use of implied correlation for arbitrage opportunities.

In the next section, we first explain two measures of implied correlations, compound and base correlations. Since we calculate base correlations in our analysis, we also present their benefits and criticism which is arising.

4.2 Measures of Implied Correlation

4.2.1 Compound Correlations

In the compound correlation approach, each tranche is considered separately, equivalently to the implied volatility approach. A pricing model is chosen and each tranche is priced using a single correlation number as input to the pricing model. By an iteration process the compound correlation can be determined as the input correlation number, which produces a spread which is equal to the market quote. A problem of this method is that it does not produce unique solutions, since the compound correlation is a function of both the upper and the lower attachment point of the tranche. This problem will be discussed in the next section.

4.2.2 Base Correlations

Base correlation is an approach proposed by JP Morgan. In opposition to compound correlations, base correlations consider the value several tranches simultaneously by applying a procedure which is called bootstrapping process by JP Morgan. According to McGinty and Ahluwalia (2004a), base correlation is defined as *the correlation inputs required for a series of equity tranches that give the tranche values consistent with quoted spreads, using the standardized* *large pool model.*

Consider a tranched credit product with upper attachment points $K_j, j = 1, ..., n$. Let $\rho_{K_j}^{BC}$ denote the base correlation of a tranche with upper attachment point K_j.The expected loss of the first equity tranche $EL_{\rho_{K_1}^{BC}}(0, K_1)$ is already traded in the market, as well as the expected losses of the non-equity tranches $EL(K_1, K_2), EL(K_2, K_3), ...$ The expected losses of the other equity tranches are calculated by a bootstrapping process:

$$
\begin{aligned}
EL_{\rho_{K_2}^{BC}}(0, K_2) &= EL_{\rho_{K_1}^{BC}}(0, K_1) + EL(K_1, K_2) \\
EL_{\rho_{K_3}^{BC}}(0, K_3) &= EL_{\rho_{K_2}^{BC}}(0, K_2) + EL(K_2, K_3)
\end{aligned}
\tag{4.1}
$$

and so on. The base correlations are the correlations, which induce these expected equity tranche losses.

4.3 Evaluation of the Base Correlation Framework

One of the main advantages of the base correlation framework over compound correlations is probably that it provides a unique implied correlation number for fixed attachment points. As they are in addition easy to understand, they became very popular. But since base correlations are a very new concept, they are also subject to discussion and criticism. Also, alternative or extending concepts of calculating implied correlations like the correlation bump (Mashal et al., 2004) are proposed in the literature.

4.3.1 Benefits of Base Correlations over Compound Correlations

In this section, we identify main benefits of base correlations over compound correlations, following McGinty and Ahluwalia (2004a) and McGinty and Ahluwalia (2004b).

Unique Correlations

The loss profile of a mezzanine tranche as illustrated in figure 2 can be interpreted in terms of a combined position in options on loss protection. Thus, a mezzanine tranche spread is not a monotonic function of correlation, but it depends on the degree, to which the two correlation sensitivities of an option long and an option short with a strike at the two attachment points offset each other. In contrast, an equity tranche spread is a monotonic function of correlation, because for the calculation of expected losses, only the upper attachment point of the tranche has to be considered. Therefore, the base correlation framework always provides a unique and more meaningful solution.

More Realistic Base Correlation Curve

Figure 3 provides a typical shape of a base correlation and of a compound correlation curve. Most credit risk models place very low probabilities on losses of senior tranches, translating into a spread of nearly zero basis points. However, since the market demands a compensation for investing in senior tranches, senior tranche spreads of a few basis points can be observed in the market. The base correlation framework accounts for these market spreads as it assigns higher correlations to senior tranches.

Valuation of Other Tranches

Base correlations provide the possibility to value tranches with arbitrary attachment points which are not traded actively relative to the traded tranches. In order to price an off-market tranche you linearly interpolate in the base correlations of the traded tranches and calculate the losses in the equity tranches corresponding to the lower and upper attachment points of the given tranche.

Relative Valuation Across Markets and Maturities

McGinty and Ahluwalia (2004b) define at-the-money (ATM) correlation as *the linearly-interpolated (or extrapolated) base correlation that matches the current expected loss of the underlying portfolio* (see for example figure 3). It provides one characteristic correlation number, which is defined equally for each portfolio. Thus, it enables to compare base correlations for products across different markets or across time. Note that it would not make sense for compound correlations due to the shape of its smile.

4.3.2 Criticism on Base Correlations

Willemann (2004) was the first to investigate the behaviour of base correlations in different settings. For generating model spreads, he uses an intensity

Fig. 3. Left figure: compound correlations as of November 3rd, 2004. Right figure: base correlations and ATM correlation as of November 3rd, 2004.

based model. He constructs an artificial portfolio of 100 names, and by choosing a single correlation (which is referred to as "intensity correlation") and identical spreads for all firms, the setup is similar to the large pool model. Further general portfolio assumptions are set equally to the example setup of JP Morgan, like a recovery rate of 40%, attachment points of the iTraxx Europe, and an interest rate of zero. In the following, we will outline some of the results.

Monotonicity of Base Correlations

Willemann (2004) calculated base correlation curves for several correlation inputs of the intensity model. Considering the two most senior tranches, it occurs that base correlation decreases with increasing intensity correlation.

First finding: from increasing intensity correlations you cannot conclude increasing base correlations with certainty.

In order to demonstrate, that the results are not due to an unnatural behaviour of the loss distribution calculated using the intensity model, two counterarguments are set forth. First, it is shown that the loss distribution is correctly skewed to the right when the intensity correlation is increased. Second, for equity tranches the intensity model produces base correlations which are monotonic in the intensity correlation. Thus, the non-monotonic relationship is due to the bootstrapping process.

Relative Valuation

In order to evaluate the relative valuation of off-market tranches as proposed in the last section, Willemann (2004) calculates for a given set of attachment points of on-market tranches the values of the off-market tranches through interpolation and directly using the intensity model.

Second finding: While the relative error for some off-market tranches is quite small, it can be quite large for more senior tranches, changing sign from tranche to tranche.

Another striking result is that for $\rho = 0$ and attachment point 12%, relative valuation produces a negative spread. We examine how this occurs. From equation 4.1 we see that the fair spread of the $(11\%, 12\%)$ tranche is solely determined by the expected losses of the equity tranches with upper attachment points 11% and 12% respectively:

$$EL(11\%, 12\%) = EL(0\%, 12\%) - EL(0\%, 11\%)$$

Now $EL(0\%, 12\%)$ is monotonically decreasing with increasing base correlation. According to the results, the slope of the correlation skew can be so steep, that the expected loss of the $(0\%, 11\%)$ tranche becomes larger than the expected loss of the $(0\%, 12\%)$ tranche and thus the expected loss of the $(11\%, 12\%)$ tranche becomes negative.

Third finding: Expected losses can go negative using relative valuation.

Uniqueness of Base Correlations

Fourth finding: Base correlations are only unique up to the set of all prior attachment points.

Recall from equation 4.1, that the base correlation of tranche (K_{j-1}, K_j) is determined by the expected losses of the equity tranches with upper attachment points K_{j-1} and K_j:

$$EL(K_{j-1}, K_j) = \underbrace{EL(0, K_j)}_{depends\ on\ K_{j-1}\ and\ K_j} - \underbrace{EL(0, K_{j-1})}_{already\ fixed}$$

Now ρ_j^{BC} and thus $EL(0, K_j)$ is fixed such that the value of tranche (K_{j-1}, K_j) is zero. In this equation one can see, that $EL(0, K_j)$ depends on K_{j-1}. The base correlation of the next tranche (K_j, K_{j+1}) in return is determined by the expected losses of the equity tranches with upper attachment points K_j and K_{j+1}:

$$EL(K_j, K_{j+1}) = EL(0, K_{j+1}) - \underbrace{EL(0, K_j)}_{fixed;\ depends\ on\ K_{j-1}\ and\ K_j}$$

Equivalently, ρ_{j+1}^{BC} and thus $EL(0, K_{j+1})$ is fixed such that the value of tranche (K_j, K_{j+1}) is zero. Therefore, ρ_{j+1}^{BC} depends via $EL(0, K_j)$ on the prior attachment point K_{j-1}.

To conclude, due to their significant advantages over compound correlations base correlations are widely used. However, as the framework is very new, there is much room for discussion, and as more data becomes available, the understanding of their characteristics and of the correlation market itself will improve.

4.3.3 An Alternative Concept: The Correlation Bump

A main shortcoming of the implied correlation approach is that quoting a single correlation number per tranche for the whole portfolio does not account for the correlation heterogeneity between the single names. A lot of information, which influences the fair value of a portfolio is neglected, thus it questionable how appropriate relative value assessment of heterogeneous portfolios based on quoted implied correlations is. The impact of this simplification can be high, which is illustrated by an example of Mashal et al. (2004).

Mashal et al. (2004) propose as an alternative measure the implied correlation bump, i.e. the number, with which all elements of the historical correlation matrix have to be multiplied to price the tranche at its current market quote. The correlation bump of a tranche which is traded at its fair value is zero. For a tranche, which is not traded at its fair value, you reprice the tranche such that the quoted prices is matched, by scaling the correlation matrix up or down. Based on for example a low scaling factor, you can make conclusions such as tranche is cheap to historical correlation, or the tranche is cheap compared to other tranches.

The advantage of this concept is, that it accounts for the heterogeneous correlation structure of the portfolio, while retaining the comfortable feature of fitting just one number. Therefore, it is a measure which is comparable across tranches. However, it is based on historical correlations, and is consequently more useful for relative value analysis than for pricing.

5 Empirical Results

In this section we provide an empirical study on the ability of CreditRisk+ to meet synthetic CDO market quotes. We make use of the fact, that the introduction of the iTraxx Europe has set a new standard in terms of liquidity and standardization in synthetic CDO products. The trading prices of the tranches provide a market view on correlation at different points in the capital structure. As the shape of the loss distribution is crucially determined by correlation, we can also deduce from this market view on correlation a market view on the shape of the loss distribution itself. We examine and compare model tranche losses and market implied tranche losses. Our goal is based on recent developments to gain insights to model calibration and to the meaning and sensitivities of tranche losses. The main question we will examine is whether it is possible to reproduce the iTraxx tranche quotes with CreditRisk+ (CR+) as a prominent credit risk model, and if so, which calibration is needed.

We will structure our analysis in three parts: in section 5.1 we set up an initial parameter calibration for both models, i.e. we define and validate initial parameter inputs, which serve as basis for further calculations. In section 5.2 we calibrate the the large pool model such that CR+ model spreads are met and we will check whether this allows us to draw conclusions for the general initial correlation level of CR+. In section 5.3 we examine how we can calibrate CR+ such that it reproduces market quotes. In the last section, we discuss briefly other ideas for calibration.

As data source we use the iTraxx Europe, the most liquidly traded credit derivative index in Europe and on which the market has also agreed to quote standard tranches. It consists of a portfolio of 125 equally weighted and most liquidly traded CDS on European companies of six sectors. It was created by the merger of the iBoxx and the TRAC-X European credit derivative indices and it was first issued on June 21st, 2004.

5.1 Setup

5.1.1 Model Setup

With the above-mentioned extensions, as input variables the CR+ model requires credit exposures, recovery rates, PDs, default rate volatilities, sector weights and sector correlations. We arbitrarily chose the November 3rd, 2004 as the basis date for our analysis [3] . For pricing issues risk neutral default probabilities (PDs) are used. Risk neutral PDs are estimated from CDS spreads

[3] We also looked at other dates, but results for other dates are very similar.

of the 125 iTraxx names as of November 3rd, 2004. The average risk neutral PD is $2.89 * 10^{-2}$. We chose an arbitrary credit exposure of $8,000$ for each credit assuming a single recovery rate of 40% for each name. Further, default rate volatilities have to be specified. In the CR+ manual in section A7.3 it is suggested that default rate volatilities are roughly equal to the PD for each name. We adopt this suggestion for our initial parameter calibration. In order to account for correlation we employ 18 different sector variables which include 17 industry sector variables and one sector variable which accounts for obligor specific risk. The sector correlation matrix is estimated from historical default data from 1994 to 2001. All estimated correlation coefficients lie in the range of 0.51 and 1.[4] For constructing sector weights, we use weights for each iTraxx name on the CreditMonitor sectors. These CreditMonitor weights are transformed to weights on our sectors by applying a sector mapping scheme [5].

For the large pool model we also assume a recovery rate of 40%. Further we assume a discount rate of 5% and quarterly spread payments. We estimate an average PD of $3.18 * 10^{-2}$ from the iTraxx index spread as of November 3rd, 2004. The PD is by nature of the base correlation model risk neutral, since it is estimated using the index quote as an average of CDS spreads. Base correlations are calculated from tranche spreads of the iTraxx as of November 3rd, 2004.

Throughout the whole analysis we have to be aware of the fact that both models used are completely different in terms of their concept and their complexity of modelling. While the CR+ model accounts for the heterogeneity of the underlying portfolio by incorporating a sector correlation matrix, a sector weight matrix and default rate volatilities, the large pool model does neither reflect any heterogeneous portfolio characteristics nor does it account for the idiosyncratic risk.

Throughout the analysis we will compare expected tranche losses, which is identical to comparing market and model quotes if the same risk neutral valuation is applied. For CR+ the full loss distribution using the mentioned input parameters is calculated analytically. Given the loss distribution as an output of CR+, expected losses for each iTraxx tranche is calculated. Absolute and percentage losses are presented in table 1. Concerning the base correlations, for each base correlation we also calculate the expected tranche loss (also presented in absolute and percentage terms in table 1), by using the large pool model and the tranche base correlation as an input of the model. That is, this

[4] It should be noted that correlations in this context are sector correlations, i.e. PD-correlations as opposed to asset correlations in structural models.

[5] Sector correlations and sector weights are not further specified here due to confidentiality

Tranche	Market Market Quotes	 Base Correlation	 Absolute Loss	 Percentage Loss	CreditRisk+, Risk Neutral PDs Absolute Loss	 Percentage Loss
(0%,3%)	24.08 [6]	20.13%	14,174.83	74.20%	14,005.47	80.73%
(3%,6%)	132.250	28.27%	2,023.41	10.59%	2,742.23	15.81%
(6%,9%)	46.000	34.70%	720.65	3.77%	496.27	2.86%
(9%,12%)	31.625	39.08%	497.41	2.60%	87.07	0.50%
(12%,22%)	15.750	49.59%	829.38	4.34%	17.98	0.10%
(22%,100%)			859.06	4.50%	0.05	0.00%
Total			19,104.74	100.00%	17,349.07	100.00%

Table 1
iTraxx Europe markets quotes and base correlations as of November 3rd, 2004; absolute and percentage losses implied by market quotes and calculated by CR+ using the initial parameter calibration.

expected tranche loss which corresponds to the tranche base correlation can be viewed to be implied by the market quote of the tranche.

For comparing the shape of the loss distribution especially in the tails of the models, we calculate the value-at-risk (VaR) at the levels 90%, 95%, 97%, 99% and 99.5%. Furthermore, we present mean, standard deviation, skewness and kurtosis. For the distributions of the large pool model, the first four sample central moments are calculated. For the distributions of CR+, the moments are calculated analytically according to Gordy (2001).

5.1.2 Initial Calibration Results

Total expected losses for CR+ using risk neutral PDs and total expected losses implied by market quotes are at a similar level. The reason therefore is that also the levels of the risk neutral PDs used in CR+ and of the average PD used in the base correlation framework is similar. Further, losses calculated by CR+ using the initial input parameters are highly concentrated on the equity tranche and on the $(3\%, 6\%)$ tranche. The equity tranche is expected to suffer 80.73% of the total expected portfolio loss, and the $(3\%, 6\%)$ tranche is expected to suffer 15.81% of the total portfolio loss, summing up to 96.53% of total losses for the two first tranches. Therefore, the left tail is very thin, expected losses for each of the three most senior tranches are 0.50% and below. Expected losses implied by the tranche spreads are much more skewed and have fatter tails. Especially the expected loss for the equity tranche of 74.20% is less than for the CR+ model. There are significant expected losses for the three most senior tranches of 2.60%, 4.34% and 4.50%.

[6] Note that the quoted 24.08 bp are an up-front payment in addition to a fixed 500.00 bp payment on the non-defaulting repayment. This explains why the equity tranche has a lower quote than the next mezzanine tranche.

The base correlations exhibit the typical observed skew. Expected losses are more evenly distributed across tranches than the CR+ losses. It is interesting to notice, that for more senior tranches the skewness and the kurtosis rise. This is consistent with the observation that for senior tranches relatively high spreads are paid. Actually expected losses of the senior tranches are nearly zero, but since nobody would take the risk for a spread of nearly zero, the expected losses implied in the market spreads are fat tailed.

5.2 Calibration of the Large Pool Model

Starting from the base case, we first examine the correlation input of the large pool model. A first focus of interest is the relationship between expected tranche losses base correlations implied by the market quotes. A second focus of interest is which base correlations we would have to observe such that the implied tranche losses meet the tranche losses of CR+ using the initial parameters. This could give us additional information on initial CR+ calibration, like on the average level of the input correlation matrix.

5.2.1 Calibration of Correlation Input

We calculate expected tranche losses and properties of the loss distribution using the large pool model with correlations of $\rho = 0, 0.2, 0.4, 0.6, 0.8$ and 0.95. We include extreme values of correlation here although they are not relevant in practice, in order to demonstrate that results are well-behaved.

Table 2 depicts results for tranche losses. For a correlation of 0, each name behaves independently from each other and the total expected percentage portfolio loss converges by the application of the law of large numbers to PD almost surely. Since the total expected portfolio loss is less than 3% of the portfolio notional, the whole expected loss is concentrated on the equity tranche. For increasing correlation mass of the loss distribution is shifted to the tails and therefore expected losses of the equity tranche decreases and expected tranche losses of the senior tranches increase. For a correlation of 0.95, the portfolio tends to behave like one asset, however expected losses are still spread out across tranches.

In table 3 VaR and the moments are given. For 0 correlation, the VaR is equal to the total expected loss and the standard deviation is approximately 0. Skewness and kurtosis do not exist for a distribution with a standard deviation of 0. For increasing correlation, due to shifting mass to the tails, standard deviation and kurtosis rise.

From this extreme example we can see the limitations of the LPGC-model when applied to relatively small portfolios. In our case we have a 125 name homogeneous portfolio with an average PD of $2.89 * 10^{-2}$. It is obvious that there is some non diversified idiosyncratic risk left because the law of large numbers does not yet fully apply. If we assume a uniform PD of $2.89 * 10^{-2}$ we learn from applying the binomial distribution that there is some 7.14% of probability mass beyond the equity piece, i.e. 7 or more losses. This result is broadly in line with the result reported in Table 6 for the case $\sigma_A = 0$. In this case, CR+ differs from the binomial distribution only by the poisson approximation error.

5.2.2 Fitting CR+ Results

A second issue we want to examine when looking at the large pool model is to calibrate the model such that we meet the CR+ expected percentage tranche losses in table 1. We start with the equity tranche and we successively solve for each tranche for its base correlation. The resulting correlation curve is depicted in figure 4. Values of the correlations range from 14.60% for the equity tranche down to 12.00% for the senior tranche. The shape of the curve is nearly flat, in opposition to the high skew of the curve from market quotes. This is a reasonable behaviour, since for CR+ we also use a single correlation matrix to calculate losses for all tranches.

Low correlations result in a distribution with low tails. Thus, the low correlation level of $12.00\% - 14.60\%$ shows again that the CR+ loss distribution has too low tails compared to market implied losses. Looking back at table 2, we can see that the CR+ tranche losses all are less than tranche losses reported for a correlation of 0.2, which confirms our results.

5.3 Calibration of the CR+ Model

Starting from the base case, we next examine input parameters of the CR+ model and try to find a calibration with which the model produces losses which are consistent with observed market quotes. Within the CR+ framework, the choice of the sector correlation matrix and of the PD volatility essentially influences the shape of the loss distribution, and both have to be quite carefully estimated in several respects.

While skewness and kurtosis of the loss distribution can only be calculated using the cumulant generating function, its standard deviation can be directly determined by the input parameters. Therefore, we look closer on the relationship between the input parameters and the standard deviation of the loss distribution. Recall equation 2.7 and the variance decomposition in the CR+ model with correlated sectors. The variance of the loss distribution is deter-

Tranche	$\rho = 0$ Absolute Loss	$\rho = 0$ Percentage Loss	$\rho = 0.2$ Absolute Loss	$\rho = 0.2$ Percentage Loss	$\rho = 0.4$ Absolute Loss	$\rho = 0.4$ Percentage Loss	$\rho = 0.6$ Absolute Loss	$\rho = 0.6$ Percentage Loss	$\rho = 0.8$ Absolute Loss	$\rho = 0.8$ Percentage Loss	$\rho = 0.95$ Absolute Loss	$\rho = 0.95$ Percentage Loss
(0%,3%)	19,104.73	100.00%	14,201.70	74.34%	10,490.24	54.91%	7,405.79	38.76%	4,574.06	23.94%	2,311.74	12.10%
(3%,6%)	0.00	0.00%	3,309.60	17.32%	3,744.59	19.60%	3,404.48	17.82%	2,667.21	13.96%	1,772.24	9.28%
(6%,9%)	0.00	0.00%	1,025.56	5.37%	1,951.05	10.21%	2,205.86	11.55%	2,015.15	10.55%	1,482.03	7.76%
(9%,12%)	0.00	0.00%	357.10	1.87%	1,133.61	5.93%	1,571.41	8.23%	1,628.00	8.52%	1,417.83	7.42%
(12%,22%)	0.00	0.00%	202.94	1.06%	1,422.79	7.45%	2,810.60	14.71%	3,794.97	19.86%	3,813.62	19.96%
(22%,100%)	0.00	0.00%	7.84	0.04%	362.44	1.90%	1,706.58	8.93%	4,425.33	23.16%	8,307.27	43.48%
Total	19,104.73	100.00%	19,104.73	100.00%	19,104.73	100.00%	19,104.73	100.00%	19,104.73	100.00%	19,104.73	100.00%

Table 2. Tranche losses of iTraxx Europe calculated by the large pool model for varying correlations. The average default probability calculated from the average index spread is $3.18 * 10^{-2}$.

		$\rho = 0$	$\rho = 0.2$	$\rho = 0.4$	$\rho = 0.6$	$\rho = 0.8$	$\rho = 0.95$
Value at Risk	90%	19,105	45,645	53,347	51,828	33,788	2,066
	95%	19,105	63,392	87,837	107,203	117,349	78,771
	97%	19,105	77,225	117,084	158,387	209,114	280,274
	99%	19,105	109,141	186,083	279,875	417,190	581,284
	99.50%	19,105	130,242	230,672	353,795	506,606	599,108
Moments	Mean	19,104.74	19,109.21	19,075.26	19,063.67	19,071.66	19,178.79
	Std Dev	0.00	22,574.63	36,815.85	52,035.36	69,869.42	89,007.66
	Skewness	-	2.81	4.05	4.82	5.21	5.31
	Kurtosis	-	15.29	25.76	31.84	32.95	31.13

Table 3. Properties of loss distribution calculated by the large pool model for varying correlations. The average default probability calculated of the average index spread is $3.18 * 10^{-2}$.

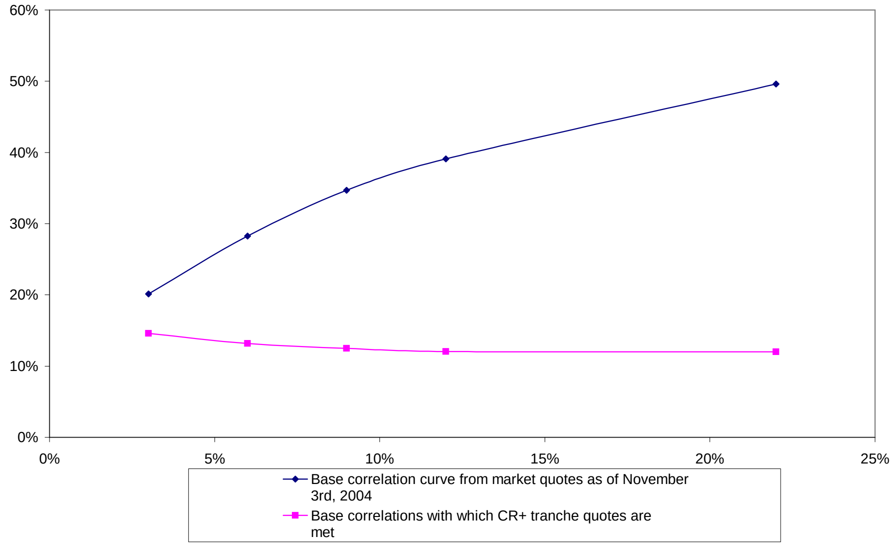

Fig. 4. Base correlation curve as of November 3rd, 2004 and base correlations with which CR+ tranche quotes are met.

mined by the unsystematic risk plus the systematic risk due to PD volatility and the systematic risk due to sector correlation and PD volatility.

In the following we will investigate the systematic risk in order to show how the variance of the loss distribution is influenced by varying correlation and PD volatility input. In addition, the standard deviation of the single factor according to equation 2.9 is reported.

Before showing empirical results, recall the effect of incorporating PD volatility and correlated sectors in the CR+ framework:

- When increasing sector correlation, holding PD volatility constant, assets are more likely to default together and the loss distribution gets more peaked with fatter tails. In equation 2.7 one can see that the variance increases with increasing sector correlation, and again increasing variance with an unchanged expected loss means fatter tails. Besides increasing correlation, also decreasing the number of sectors increases the variance, since less sectors mean more concentrated risk. Thus, sector correlation contributes most to the variance of the loss distribution, if all correlation coefficients are set equal to 1, which is equivalent to incorporating only a single sector.
- Similarly, the effect of incorporating PD volatility can be explained. When incorporating PD volatility, there is more chance to experience extreme losses, meaning that the tails of the distribution become fatter. Take for example a VaR at 99%. If PD volatility is increased, the probability that total losses exceed 99% rises. This can also be illustrated by considering equation 2.7. When setting $\sigma_k = 0$ for all k, only the first sum is left. When

introducing PD volatility, the variance of the loss distribution increases. An increased variance, while remaining expected losses unchanged, means that the tails become fatter.

This shows, that we can try to fit CR+ tranche losses to the fatter tailed tranche losses implied by model quotes by increasing sector correlation and PD volatility.

5.3.1 Calibration of the Correlation Input

We vary the original correlation matrix by multiplying it by a chosen factor and then adding or subtracting a constant, such that off-diagonal elements are varied while diagonal elements remain equal to 1. Thus, the new correlation matrix still accounts for different levels of correlation coefficients within the original matrix.

Tranche losses for varying correlation are shown in table 4. For zero correlation most of the expected losses concentrate on the equity tranche, however, due to the unsystematic risk and the risk due to PD volatility the next two more senior tranches are also expected to suffer small losses. Considering tranche losses for increasing correlation, losses are shifted from the equity tranche to more senior tranches. But even in the case of a correlation of 1, still 95.24% of total expected losses concentrate on the equity and the $(3\%, 6\%)$ tranche. Thus, compared to the results for the large pool model, increasing correlation does not result in equally heavy tails here.

Properties of the loss distributions are shown in table 5. For 0 correlation, VaRs for the different levels are very low and they lie close together. Also standard deviation is quite low. VaRs are not equal to each other and standard deviation is not 0 as in the large pool model with 0 correlation due to the unsystematic risk and the risk due to PD volatility. Using a single factor, VaRs become more spread out due to the increased standard deviation of the distribution. For middle values of correlation, standard deviation and thus kurtosis rise. It can be seen that the increase of standard deviation is only due to an increase of the systematic risk due to correlation.

5.3.2 Calibration of the PD Volatility Input

Initially, PD volatility σ_A was set equal to $1 * p_A$. Now we vary PD volatility $\sigma_A = f_{\sigma_A} * p_A$ by using factors from $f_{\sigma_A} = 0$ to $f_{\sigma_A} = 3$. Tranche losses and properties of the loss distribution are shown in tables 6 and 7. [7]

[7] CR+ assumes gamma distributed PDs. It should be noted that by increasing the standard deviation of the gamma distribution beyond its expected value its shape

	zero correlation		$\rho = 2 * \rho_{org} - 1$		$\rho = 1.5 * \rho_{org} - 0.5$		$\rho_{org}(\ basecase)$		$\rho = 0.5 * \rho_{org} + 0.5$		$singlefactor$	
Tranche	Absolute Loss	Percentage Loss	Absolute Loss	Percentage Loss	Absolute Loss	Percentage Loss	Absolute Loss	Percentage Loss	Absolute Loss	Percentage Loss	Absolute Loss	Percentage Loss
(0%,3%)	16,503.93	95.13%	14,493.56	83.54%	14,243.88	82.10%	14,005.47	80.73%	13,777.53	79.41%	13,559.32	78.16%
(3%,6%)	839.41	4.84%	2,459.88	14.18%	2,609.57	15.04%	2,742.23	15.81%	2,859.98	16.48%	2,964.63	17.09%
(6%,9%)	5.72	0.03%	344.53	1.99%	420.12	2.42%	496.27	2.86%	571.88	3.30%	646.14	3.72%
(9%,12%)	0.01	0.00%	44.78	0.26%	64.33	0.37%	87.07	0.50%	112.55	0.65%	140.32	0.81%
(12%,22%)	0.00	0.00%	6.32	0.04%	11.16	0.06%	17.98	0.10%	27.02	0.16%	38.42	0.22%
(22%,100%)	0.00	0.00%	0.01	0.00%	0.02	0.00%	0.05	0.00%	0.12	0.00%	0.24	0.00%
Total	17,349.07	100.00%	17,349.07	100.00%	17,349.07	100.00%	17,349.07	100.00%	17,349.07	100.00%	17,349.07	100.00%

Table 4. Tranche losses of iTraxx Europe calculated by CR+ for varying correlation matrices.

		zero correlation	$\rho = 2 * \rho_{org} - 1$	$\rho = 1.5 * \rho_{org} - 0.5$	$\rho_{org}(\ basecase)$	$\rho = 0.5 * \rho_{org} + 0.5$	$singlefactor$
Value at Risk	90%	33,600	38,400	38,400	43,200	43,200	43,200
	95%	38,400	48,000	52,800	52,800	52,800	57,600
	97%	38,400	57,600	62,400	62,400	62,400	67,200
	99%	48,000	76,800	76,800	81,600	86,400	86,400
	99.50%	52,800	86,400	91,200	91,200	96,000	100,800
Moments	Mean	17,349	17,349	17,349	17,349	17,349	17,349
	Std Dev	10,443	16,771	17,522	18,242	18,934	19,603
	Skewness	0.74	1.65	1.75	1.84	1.93	2.01
	Kurtosis	3.73	7.03	7.54	8.04	8.55	9.06
Std Dev of Single Factor		0.29	0.81	0.86	0.91	0.96	1.00
Systematic Risk due to PD Volatility		25,771,355	25,771,355	25,771,355	25,771,355	25,771,355	25,771,355
Systematic Risk due to PD Volatility and Sector Correlation		0	172,209,945	197,962,589	223,715,232	249,467,876	275,220,520

Table 5. Properties of loss distribution calculated by CR+ for varying correlation matrices.

Results for tranche losses essentially suggest the same qualitative conclusions as for varying correlation. For increasing PD volatility tranche losses are shifted from the equity tranche to more senior tranches, however the impact is much stronger. For $\sigma_A = 0$ almost all expected losses are concentrated on the equity tranche. For increasing $\sigma_A = p_A * 3$ only 40.47% of total losses remain for the equity tranche and losses for three senior tranches are quite high. We can see that by varying PD volatility we can obtain heavy tails as of the tranche losses implied by the market quotes. However, in order to meet the tranche losses implied by market quotes we have to use a different PD volatility vector for each tranche. The factors for which the tranche losses implied by market quotes are exactly met are $f_{\sigma_A} = 1.25, 0.68, 1.10, 1.48, 1.88, 2.71$ for each tranche, ordered by seniority. These results could be described as a "PD volatility skew", of which the shape reminds to the compound correlation skew. In fact, this skew exists due to the same reasons as the correlation skew.

For $\sigma_A = 0$ there is no systematic risk and thus the standard deviation of the single factor is 0. The variance of the loss distribution is only due to the unsystematic risk. For increasing PD volatility, the variance increases, meaning that while expected losses only decrease slightly, that tails become fatter. In contrast to the results for increased correlation, we can observe a considerable increase of the variance and the kurtosis for high PD volatilities. The reason is, that for a high PD volatility as for example for $\sigma_A = p_A * 3$, the last sum in equation 2.7 is 9 times higher than in the base case, while for an increased variance the last sum is only about 2 times higher. Therefore we have senior tranche losses which are high enough to meet the market implied tranche losses.

5.3.3 Other Calibrations

So far, we examined results starting with the base case and varying only one input variable. In addition, we also looked at other calibrations varying both PD volatility and correlation at the same time. For example, using a PD volatility input of $\sigma_A = 1 * p_A$, even for increasing correlation up to 1 we did not reach the heavy tails implied by market quotes. Therefore, we calculated tranche losses using the same correlation inputs, while setting the PD volatility on a higher level. For example for $\sigma_A = 3 * p_A$ we can observe that we can meet market implied tranche losses using different correlations, which produces a correlation skew similar to the before observed PD volatility skew.

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degenerates to a monotonously decreasing function with density equal to infinity at 0. This is because probability mass has to be shifted towards 0 to compensate higher PD realizations necessary to increase PD-volatility while maintaining the unconditional average PD. It has been argued that such a parametrization contradicts economic intuition.

Tranche	$\sigma_A = 0$ Absolute Loss	$\sigma_A = 0$ Percentage Loss	$\sigma_A = p_A * 1$ Absolute Loss	$\sigma_A = p_A * 1$ Percentage Loss	$\sigma_A = p_A * 1.5$ Absolute Loss	$\sigma_A = p_A * 1.5$ Percentage Loss	$\sigma_A = p_A * 2$ Absolute Loss	$\sigma_A = p_A * 2$ Percentage Loss	$\sigma_A = p_A * 2.5$ Absolute Loss	$\sigma_A = p_A * 2.5$ Percentage Loss	$\sigma_A = p_A * 3$ Absolute Loss	$\sigma_A = p_A * 3$ Percentage Loss
(0%,3%)	16,844.24	97.09%	14005.47	80.73%	11773.25	67.86%	9809.57	56.55%	8205.72	47.42%	6919.32	40.47%
(3%,6%)	504.42	2.91%	2742.23	15.81%	3554.92	20.49%	3750.70	21.62%	3630.17	20.98%	3375.24	19.74%
(6%,9%)	0.41	0.00%	496.27	2.86%	1258.67	7.25%	1792.12	10.33%	2038.70	11.78%	2090.48	12.23%
(9%,12%)	0.00	0.00%	87.07	0.50%	468.59	2.70%	916.42	5.28%	1228.74	7.10%	1387.27	8.11%
(12%,22%)	0.00	0.00%	17.98	0.10%	280.24	1.62%	924.85	5.33%	1658.79	9.59%	2229.76	13.04%
(22%,100%)	0.00	0.00%	0.05	0.00%	13.41	0.08%	153.51	0.88%	543.38	3.14%	1093.83	6.40%
Total	17349.07	100.00%	17349.07	100.00%	17349.07	100.00%	17347.17	100.00%	17305.49	100.00%	17095.91	100.00%

Table 6. Tranche losses of iTraxx Europe calculated by CR+ for varying PD volatilities.

		$\sigma_A = 0$	$\sigma_A = p_A * 1$	$\sigma_A = p_A * 1.5$	$\sigma_A = p_A * 2$	$\sigma_A = p_A * 2.5$	$\sigma_A = p_A * 3$
Value at Risk	90%	28800	43,200	48,000	52,800	52,800	52,800
	95%	33600	52,800	67,200	81,600	91,200	100,800
	97%	38400	62,400	81,600	105,600	124,800	139,200
	99%	43200	81,600	115,200	158,400	201,600	240,000
	99.50%	43200	91,200	139,200	192,000	249,600	312,000
Moments	Mean	17,349	17,349	17,349	17,349	17,349	17,349
	Std Dev	9,126	18,242	25,389	32,882	40,529	48,256
	Skewness	0.53	1.84	2.74	3.64	4.55	5.46
	Kurtosis	3.28	8.04	14.23	22.92	34.10	47.77
Std Dev of Single Factor		0.00	0.91	1.37	1.82	2.28	2.73
Systematic Risk due to PD Volatility		0.00	25,771,355	57,985,549	103,085,420	161,070,969	231,942,195
Systematic Risk due to PD Volatility and Sector Correlation		0.00	223,715,232	503,359,273	894,860,930	1,398,220,203	2,013,437,095

Table 7. Properties of loss distribution calculated by CR+ for varying PD volatilities.

Up to now, we were interested in the fit of single tranche losses. Beside this, we also examined which model input minimizes the sum of absolute tranche loss deviations

$$min_{f_\rho, f_{\sigma_A}} \sum_i \mid EL_{BC}(K_i, K_{i+1}) - EL_{CR+}(K_i, K_{i+1}) \mid$$

where f_ρ and f_σ are the factors with which correlation and PD volatilities are manipulated in the way explained above. The solution to this minimization problem is $f_\rho = 1$ and $f_{\sigma_A} = 1.050$, which produces tranche losses of 76.72%, 17.72%, 4.22%, 1.01%, 0.32% and 0.00%. Interestingly, the risk due to correlation is risen up to its maximum. However by minimizing the sum loss deviation over all tranches, a completely different goal is pursued than in the last section. Here no attention is paid to the shape of the distribution, as the thickness of the tails is neglected.

6 Conclusions

In this paper we investigated the ability of CreditRisk+ to reproduce market quotes of the iTraxx Europe. We first review correlation modeling in the most prominent credit risk models by the industry, CreditMetrics and CR+. We also describe the homogeneous large pool Gaussian copula model that is often used for CDO evaluation. Then we summarize approaches to implied correlations for CDO tranches including the compound correlation and base correlation framework. In our empirical analysis, we find that using initial parameters, CR+ produces much less heavy tails compared to the losses of the senior tranches traded in the market. In a second step the large pool model was calibrated such that the CR+ model quotes are met. The resulting shape of the correlation curve is nearly flat and the range of values is low compared to the curve from market quotes. Both results show that in order to reproduce market quotes, parameters of CR+ have to be adjusted such that a loss distribution with fatter tails is produced.

In a third step two possible adjustments of CR+ parameters were analyzed, an increase of the sector correlation matrix and an increase of the PD volatility. Empirical results show that the sensitivity of senior tranche losses to sector correlation is low and even by increasing correlation up to 1 it is not possible to reproduce senior tranche losses observed in the market. However, by increasing the PD volatility input for each name CR+ produces tails which are fat enough to meet market tranche losses. Also, in order to fit each tranche loss, a different PD volatility vector has to be used. These results can be interpreted as being similar to the idea of correlation skew in the large pool Gaussian copula model.

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