Momentum Strategies Based on Reward-Risk Stock Selection Criteria

Svetlozar Rachev*

Chair-Professor, Chair of Econometrics, Statistics
and Mathematical Finance
School of Economics and Business Engineering
University of Karlsruhe,
Postfach 6980, 76128 Karlsruhe, Germany
and
Department of Statistics and Applied Probability
University of California, Santa Barbara
CA 93106-3110, USA
E-mail: rachev@statistik.uni-karlsruhe.de

Teo Jašić

Institute of Econometrics, Statistics
and Mathematical Finance
School of Economics and Business Engineering
University of Karlsruhe
Kollegium am Schloss, Bau II, 20.12, R210
Postfach 6980, D-76128, Karlsruhe, Germany
E-mail: teo.jasic@gmx.de

Stoyan Stoyanov

Chief Financial Researcher
FinAnalytica Inc.
130 S. Patterson
P.O. Box 6933
CA 93160-6933, USA
E-mail: stoyan.stoyanov@finanalytica.com

Frank J. Fabozzi

Adjunct Professor of Finance and Becton Fellow,
Yale School of Management,
135 Prospect Street, Box 208200,
New Haven, Connecticut 06520-8200 U.S.A.
E-mail: fabozzi321@aol.com

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ABSTRACT

In this paper, we analyze momentum strategies that are based on reward-risk stock selection criteria in contrast to ordinary momentum strategies based on a cumulative return criterion. Reward-risk stock selection criteria include the standard Sharpe ratio with variance as a risk measure, and alternative reward-risk ratios with the expected shortfall as a risk measure. We investigate momentum strategies using 517 stocks in the S&P 500 universe in the period 1996 to 2003. Although the cumulative return criterion provides the highest average monthly momentum profits of 1.3% compared to the monthly profit of 0.86% for the best alternative criterion, the alternative ratios provide better risk-adjusted returns measured on an independent risk-adjusted performance measure. We also provide evidence on unique distributional properties of extreme momentum portfolios analyzed within the framework of general non-normal stable Paretian distributions. Specifically, for every stock selection criterion, loser portfolios have the lowest tail index and tail index of winner portfolios is lower than that of middle deciles. The lower tail index is associated with a lower mean strategy. The lowest tail index is obtained for the cumulative return strategy. Given our data-set, these findings indicate that the cumulative return strategy obtains higher profits with the acceptance of higher tail risk, while strategies based on reward-risk criteria obtain better risk-adjusted performance with the acceptance of the lower tail risk.

Key words: momentum strategies, reward-risk stock selection criteria, expected tail loss, stable Paretian distribution, risk-adjusted performance

A number of studies document the profitability of momentum strategies across different markets and time periods (Jegadeesh and Titman, 1993, 2001, Rouwenhorst 1998, Griffin *et al*. 2003). The strategy of buying past winners and selling past losers over the time horizons between 6 and 12 months provides statistically significant and economically large payoffs with historically earned profits of about 1% per month. The empirical evidence on the momentum effect provides a serious challenge to asset pricing theory. There is so far no consistent risk-based explanation and, contrary to other financial market anomalies such as the size and value effect that gradually disappear after discovery, momentum effect persists.

Stock selection criteria play a key role in momentum portfolio construction. While other studies apply simple cumulative return or total return criterion using monthly data, we apply reward-risk portfolio selection criteria to individual securities using daily data. A usual choice of reward-risk criterion is the ordinary Sharpe ratio corresponding to the static mean-variance framework. The mean-variance model is valid for investors if (1) the returns of individual assets are normally distributed or (2) for a quadratic utility function, indicating that investors always prefer portfolio with the minimum standard deviation for a given expected return. Either one of these assumptions are questionable. Regarding the first assumption, there is overwhelming empirical evidence that invalidates the assumption of normally distributed asset returns since stock returns exhibit asymmetries and heavy tails. In addition, further distributional properties such as known kurtosis and skewness are lost in the one-period mean-variance approach.

Various measures of reward and risk can be used to compose alternative reward-risk ratios. We introduce alternative risk-adjusted criteria in the form of reward-risk ratios that use the expected shortfall as a measure of risk and expectation or expected shortfall as a measure of reward. The expected shortfall is an alternative to the value-at-risk (VaR) measure that overcomes the limitations of VaR with regards to the properties of coherent risk measures (Arztner et al., 1999). The motivation in using alternative risk-adjusted criteria is that they may provide strategies that obtain the same level of abnormal momentum returns but are less risky than those based on cumulative return criterion.

In previous and contemporary studies of momentum strategies, possible effects of non-normality of individual stock returns, their risk characteristics, and the distributional properties of obtained momentum datasets have not received much attention. Abundant empirical evidence shows that individual stock returns exhibit non-normality, leptokurtic, and heteroscedastic properties which implies that such effects are clearly important and may have a considerable impact on reward and riskiness of investment strategies. When the return distribution is heavy tailed, extreme returns occur with a much larger probability than in the case of the normal distribution. In addition, quantile-based measures of risk, such as VaR, may also be significantly different if calculated for heavy-tailed distributions. As shown by Tokat et al. (2003), two distributional assumptions (normal and stable Paretian[1]) may result in considerably different asset allocations depending on the objective function and the risk-aversion level of the decision maker. By using the risk measures that pay more attention to the tail of the distribution, preserving the heavy tails with the use of a stable model makes an important difference to the investor who can earn up to a multiple of the return on the unit of risk he bears by applying the stable model. Thus, consideration of a non-normal return distribution plays an important role in the evaluation of the risk-return profile of individual stocks and portfolios of stocks.

Additionally, the distributional analysis of momentum portfolios obtained on some stock ranking criteria provides insight in what portfolio return distribution the strategy generates. The evidence on the distributional properties of momentum datasets in the contemporary literature is only fragmentary. Harvey and Siddique (2000) analyzed the relation between the skewness and the momentum effect on the momentum datasets formed on cumulative return criterion. They examine cumulative return strategy on NYSE/AMEX and Nasdaq stocks with five different ranking (i.e., 35 months, 23 months, 11 months, 5 months and 2 months) and six holding periods (i.e., 1 month, 3 months, 6 months, 12 months, 24 months and 36 months) over the period January 1926 to December 1997. Their results show that for all momentum strategy definitions, the

[1] The observation by Mandelbrot (1963) and Fama (1963, 1965) of excess kurtosis in empirical financial return processes led them to reject the normal distribution assumption and propose non-Gaussian stable processes as a statistical model for asset returns. Non-Gaussian stable distributions are commonly referred to as “stable Paretian” distribution due to the fact that the tails of the non-Gaussian stable density have Pareto power-type decay.

skewness of the loser portfolio is higher than that of the winner portfolio. They conclude that there exists a systematic skewness effect across momentum portfolio deciles in that the higher mean strategy is associated with lower skewness. Although we consider a much shorter dataset than Harvey and Siddique, our conclusions are similar for an extended set of alternative reward-risk stock selection criteria.

We extend the distributional analyses in that we estimate the parameter of the stable Paretian distribution and examine the non-normal properties of the momentum deciles. Stable Paretian distributions are a class of probability laws that have interesting theoretical and practical properties. They generalize the normal (Gaussian) distribution and allow heavy tails and skewness, which are frequently seen in financial data. Our evidence shows that the loser portfolios have the lowest tail index for every criterion, and that the tail index of the winner portfolio is higher than that of loser portfolio for every criterion. In addition, we also find a systematic skewness pattern across momentum portfolios for all criteria, with the sign and magnitude of the skewness differential between loser and winner portfolios dependent on the threshold parameter in reward-risk criteria. We interpret these findings as evidence that extreme momentum portfolio returns have non-normal distribution and contain additional risk component due to heavy tails. The part of momentum abnormal returns may be compensation for the acceptance of the heavy-tailed distributions (with the tail index less than that of the normal distribution) and negative skewness differential between winner and loser portfolios.

We examine our alternative strategies based on various reward-risk criteria on the sample of 517 S&P 500 firms over the January 1996 – December 2003 period. The largest monthly average returns are obtained for cumulative return criterion. We also evaluate the performance of different criteria using a risk-adjusted independent performance measure which takes the form of reward-risk ratio applied to resulting momentum spreads. On this measure, the best risk-adjusted performance is obtained using the best alternative ratio followed by the cumulative return criterion and the Sharpe ratio. Following our analysis, we argue that risk-adjusted momentum strategy using alternative ratios provides better risk-adjusted returns than cumulative return criterion although it may provide profits of lower magnitude than those obtained using cumulative return criterion. Regarding the comparison of performance among various reward-risk criteria, we find that all alternative criteria obtain better risk-adjusted performance than the Sharpe ratio for our momentum strategy. A likely reason is that the alternative ratio criteria capture better the non-normality properties of individual stock returns than the traditional mean-variance measure of the Sharpe ratio. An important implication of these results concerns the concept of risk measure in that the variance as a dispersion measure is not appropriate where the returns and non-normal and that the expected tail loss measure focusing on tail risk is a better choice.

The remainder of the paper is organized as follows. Section 1 provides a definition of risk-adjusted criteria as alternative reward-risk ratios. Section 2 describes the data and methodology. Section 3 conducts distributional analysis of the momentum portfolio daily returns obtained using applied criteria and evaluates the performance of resulting momentum strategies on an independent risk-adjusted performance measure. Section 4 concludes the paper.

1. Risk-Adjusted Criteria for Stock Ranking

The usual approach to selecting winners and losers employed in previous and contemporary studies on momentum strategies has been to evaluate the individual stock's past monthly returns over the ranking period (e.g., six-month monthly return for the six-month ranking period). The realized cumulative return as a selection criterion is a simple measure, which does not include the risk component of the stock behavior in the ranking period[2]. To consider a risk component of an individual stock, we may apply different risk measures to capture the risk profile of stock returns. Variance or volatility of the stock

[2] In mean-variance space, winners are positioned on the upper part of the efficient mean-variance frontier, so that by construction they entail the highest risk Losers simply correspond to the winners multiplied by -1. In the observed universe of assets bounded by the mean-variance frontier, there may be riskier assets with higher variance (measured by standard deviation) than those of selected winners (or losers) but they will not qualify as winner or loser based on cumulative return criterion. Similar to return ranking criterion, we might think of applying a variance ranking criterion. However, the stocks with the greatest variance do not necessarily need to be the stocks with the greatest return at the same time and would not be on the efficient frontier.

returns captures the riskiness of the stock with regards to the traditional mean-variance framework. The disadvantage of variance is that the investor's perception of risk is assumed to be symmetric around the mean. However, we can consider other risk measures that overcome the deficiencies of the variance and satisfy the properties of coherent risk measures. In addition, empirical evidence shows that individual stock returns exhibit non-normality, so it would be more reliable to use a measure that could account for these properties.

Considering the non-normal properties of stock returns in the context of momentum trading, we aim to obtain risk-adjusted performance criterion that would be applicable to the most general case of a non-Gaussian stable distribution of asset returns[3]. It is a well established fact based on empirical evidence that asset returns are not normally distributed, yet the vast majority of the concepts and methods in theoretical and empirical finance assume that asset returns follow a normal distribution. Since the initial work of Mandelbrot (1963) and Fama (1963; 1965) who rejected the standard hypothesis of normally distributed returns in favor of a more general stable Paretian distribution, the stable distribution has been applied to modeling both the unconditional and conditional return distributions, as well as theoretical framework of portfolio theory and market equilibrium models (see Rachev, 2003). While the stable distributions are stable under addition (i.e., a sum of stable independent and identically distributed (i.i.d.) random variables is a stable random variable), they are fat-tailed to the extent that their variance and all higher moments are infinite.

We introduce next the expected shortfall as a measure of risk that will be used as the risk component of the alternative reward-risk criteria.

1.1 Expected Shortfall

Usual measures of risk are standard deviation and VaR. The VaR at level $(1-\alpha)100\%$, $\alpha \in [0,1]$, denoted $VaR_{(1-\alpha)100\%}(r)$ for an investment with random return r, is defined by $Pr(l$

[3] This choice can be further complicated by considering which risk-adjusted criteria are more useful for an investor rather than another.

$> VaR_{(1-\alpha)100\%}(r)) = \alpha$, where $l = -r$ is the random loss, that can occur over the investment time horizon. In practice, values of α close to zero are of interest, with typical values of 0.05 and 0.01. VaR is not "sensitive" to diversification and, even for sums of independent risky positions, its behavior is not as we would expect (Fritelli and Gianin, 2002). The deficiencies of the VaR measure prompted Arztner et al. (1999) to propose a set of properties any reasonable risk measure should satisfy. They introduce the idea of coherent risk measures, with the properties of monotonicity, sub-additivity, translation invariance, and positive homogeneity.

Standard deviation and VaR are not coherent measures of risk. In general, VaR is not subadditive and is law invariant in a very strong sense. On the other hand, *expected shortfall* is a coherent risk measure (Arztner et al., 1999; Rockafellar and Uryasev, 2002, Bradley and Taqqu, 2003). It is also called *expected tail loss* (ETL) or *conditional VaR*[4] (CVaR). ETL is a more conservative measure than VaR and looks at how severe the average (catastrophic) loss is if VaR is exceeded[5]. Formally, ETL is defined by

$$ETL_{\alpha 100\%}(r) = E(l|l > VaR_{(1-\alpha)100\%}(r)), \qquad (1)$$

where r is the return over the given time horizon, and $l = -r$ is the loss. $ETL_{\alpha 100\%}(r)$ is, denoted by $CVaR_{(1-\alpha)100\%}(r) = ETL_{\alpha 100\%}(r)$ (see Martin *et al*. 2003).

ETL is a subadditive, coherent risk measure and portfolio selection with the expected shortfall can be reduced to a linear optimization problem (see Martin *et al*. 2003, and the references therein).

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[4] There are some notational inconsistencies in the literature on expected shortfall. Some authors distinguish between expected tail loss and CVaR, so that only in the case of continuous random variable the definition of expected shortfall coincides with that of CVaR.

[5] Expected shortfall conveys the information about the expected size of a loss exceeding VaR. For example, suppose that a portfolio's risk is calculated through simulation. For 1,000 simulations and α= 0.95, the portfolio's VaR would be the smallest of the 50 largest losses. The corresponding expected shortfall would be then estimated by the numerical average of these 50 largest losses.

1.2 Alternative Reward-Risk Ratios

Recently, Biglova et al. (2004) provide an overview of various reward-risk performance measures and ratios that have been studied in the literature and compare them based on the criterion of maximizing the final wealth over a certain time period. The results of the study support the hypothesis that alternative risk-return ratios based on the expected shortfall capture the distributional behavior of the data better than the traditional Sharpe ratio. In order to include the risk profile assessment and account for non-normality of asset returns, we apply the alternative Stable-Tail Adjusted Return ratio (STARR ratio) and the Rachev ratio (R-ratio)[6] as the criteria in forming momentum portfolios. We analyze and compare the traditional Sharpe ratio with alternative STARR and R-ratios for various parameter values that define different level of coverage of the tail of the distribution.

1.2.1 Sharpe ratio

The *Sharpe ratio* (Sharpe, 1994) is the ratio between the expected excess return and its standard deviation:

$$\rho(r) = \frac{E(r - r_f)}{\sigma_{(r-r_f)}} = \frac{E(r - r_f)}{\sigma_r} \qquad (2)$$

where r_f is the risk-free asset and σ_r is the standard deviation of r. For this ratio it is assumed that the second moment of the excess return exists. In the investment applications, the decisions based on Sharpe ratio are compatible with Gaussian returns or, in general, with elliptically distributed returns with finite second moments. However, erroneous asset selection decisions may be made when the Sharpe ratios are applied to asset returns that follow a non-Gaussian distribution (Bernardo and Ledoit, 2000).

1.2.2 STARR ratio

[6] R-ratio stands for Rachev ratio and was introduced in the context of risk-reward alternative performance measures and risk estimation in portfolio theory (Biglova, Ortobelli, Rachev, and Stoyanov, 2004).

The $STARR_{(1-\alpha)100\%}$ *ratio* ($CVaR_{(1-\alpha)100\%}$ *ratio*) is the ratio between the expected excess return and its conditional value at risk (Martin *et al*., 2003):

$$\text{STARR}((1-\alpha)\cdot 100) := \text{STARR}_{(1-\alpha)100\%} := \rho(r) = \frac{E(r-r_f)}{CVaR_{(1-\alpha)100\%}(r-r_f)} \tag{3}$$

where $CVaR_{(1-\alpha)100\%}(r) = ETL_{\alpha 100\%}(r)$, see (1). The STARR ratio can be evaluated for different levels of the parameter α that represents the significance level α of the left tail of the distribution. A choice of STARR ratios with different significance level α indicates different levels of consideration for downside risk and, in a certain sense, can represent the different levels of risk-aversion of an investor. For example, the use of STARR ratio with larger values of α (e.g., 0.4, 0.5) resembles behavior of a more risk-averse investor that considers a large portion or complete downside risk compared to low values of α (e.g., 0.1, 0.05) representing less risk averse investor.

1.2.3 Rachev ratio

The *Rachev ratio* (*R-ratio*) with parameters α and β is defined as:

$$\text{RR}(\alpha, \beta) := \text{RR}_{(\alpha, \beta)} := \rho(r) = \frac{ETL_{\alpha 100\%}(r_f - r)}{ETL_{\beta 100\%}(r - r_f)} \tag{4}$$

where α and β are in [0,1]. Here, if r is a return on a portfolio or asset, and $ETL_\alpha(r)$ is given by (1). Thus, the R-ratio is the ratio of the ETL of the opposite of excess return at a given confidence level, divided by the ETL of the excess return at another confidence level. The R-ratio is applied for different parameters α and β. For example, R-ratio ($\alpha = 0.01, \beta = 0.01$), R-Ratio ($\alpha = 0.05, \beta = 0.05$), and R-ratio ($\alpha = 0.09, \beta = 0.9$). The parameters α and β cover different significance levels of the right and left tail distribution, respectively.

The concept of the R-ratio construct mimics the behavior of a savvy investor who aims to simultaneously maximize the level of return and gets insurance for the maximum loss. The R-ratio as given by (4) can be interpreted as the ratio of the expected tail return above a certain threshold level (100 α-percentile of the right tail distribution), divided by the expected tail loss beyond some threshold level (100 β-percentile of the right tail distribution). The part of the distribution between the significance levels α and β is not taken into consideration. In other words, the R-ratio that awards extreme returns adjusted for extreme losses. Note that the STARR ratio is the special case of the R-ratio since $CVaR_{(1-\alpha)100\%} = ETL_{\alpha 100\%}$. For example, STARR(95%) = R-ratio(1, 0.05). When the parameters in the R-ratio are used in percentage form, they correspond to the $(1 - \alpha)$ confidence level notation to ease the comparison with the STARR ratios (i.e., R-ratio((1-α)·100%,(1-β)·100%)).

The distinctive feature of alternative STARR and R-ratio is that they assume only finite mean of the return distribution and require no assumption on the second moment. Thus, alternative ratios can evaluate return distributions of individual stocks that exhibit heavy tails using the ETL measure. In contrast, the Sharpe ratio is defined for returns having a finite second moment. Moreover, the choice of different tail probabilities for the parameters of alternative ratios enables modeling different levels of risk aversion of an investor.

2. Formation of Momentum Portfolios using Reward-Risk Stock Selection Criteria

The data sample consists of a total of 517 stocks included in the S&P 500 index in the period January 1, 1996 to December 31, 2003. Daily stock returns in the observed period were calculated as

$$r_{i,t} = \ln \frac{P_{i,t}}{P_{i,t-1}}$$

where $r_{i,t}$ is the return of the i-th stock at time t and $P_{i,t}$ is the (dividend adjusted) stock price of the i-th stock at time t.

The momentum strategy is implemented by simultaneously selling losers and buying winners at the end of the formation or ranking period, and holding the portfolio over the investment or holding period. The winners and losers in the ranking period are determined using stock selection criterion that evaluates prior individual returns of all available stocks in the ranking period. Since the dollar amount of the long portfolio matches that amount shorted in the short portfolio, the momentum strategy is a zero-investment, self-financing strategy that may generate momentum profits in the holding period. A zero-investment strategy is applicable in international equity investment management practice given the regulations on proceeds from short-sales for investors.

We consider momentum strategies based on the ranking and holding periods of 6 months (i.e., 6-month/6-month strategy or shortly 6/6 strategy). We rank individual stocks by applying the cumulative return, Sharpe ratio and alternative STARR and R-ratio criteria. The STARR ratios include $STARR_{99\%,}$ $STARR_{95\%,}$ $STARR_{90\%,}$ $STARR_{75\%,}$ and $STARR_{50\%}$ criteria that model increasing levels of risk-aversion of an investor, respectively. The R-ratio criteria include R-ratio(99%,99%) (i.e., R-ratio(0.01,0.01), R-ratio(95%,95%), R-ratio(91%,91%), R-ratio(50%,99%), and R-ratio(50%,95%) that model different sensitivity levels to large losses and large profits.

The chosen criteria are applied to daily returns of individual stocks in the ranking period of 6 months. Therefore, for each month *t*, the portfolio held during the investment period, months t to $t + 5$, is determined by performance over the ranking period, months $t - 6$ to $t - 1$. Following the usual convention, the stocks are ranked in ascending order and assigned to one of the ten deciles (sub-portfolios). “Winners” are the top decile (10%) of the stocks with the highest values of the stock selection criteria in the ranking periods, and “losers” are the bottom decile (10%) of all stocks with the lowest values of criteria in the ranking period. All the stocks satisfy the requirement that their returns exist at least 12 months before applying the risk-return criterion in the first ranking period. Winner and loser portfolios are equally weighted at formation and held for 6 subsequent months of the holding period. The strategy is applied to non-overlapping 6-month investment horizons, so that the positions are held for 6-months, after which the portfolio is re-constructed (rebalanced).

After forming the combined portfolio of winners and losers, we evaluate the performance of the momentum strategy in the holding period. Specifically, we analyze average momentum (winner minus loser) spread returns, cumulative profits (final wealth) of the momentum portfolio and the risk-adjusted performance. These performance measures are evaluated using daily returns. Following the analysis of momentum profits and risk-adjusted performance, we identify the best performing ratios, which allow investors to pursue a profitable and risk-balanced momentum strategy. Each class of ratio criteria represents a statistical arbitrage that follows a defined reward-risk profile.

3. Results

3.1 Summary Statistics of Momentum Portfolio Returns

For every stock-selection criterion, average returns and summary statistics based on daily data are reported[7]. For the convenience of comparison with the results of other studies on the 6/6 strategy using cumulative return criterion, we calculate the *t*-statistics[8] for average daily spread returns. The first column of Table 1 shows the average daily returns of winner and loser portfolios as well as of the zero-cost, winner-loser spread portfolios for our 6-month/6-month strategy for all considered reward-risk ratios and cumulative return criterion. The largest average winner-loser spread of 0.061% per day (an associated *t*-statistic of 1.55) or 1.28% per month for the 6-month/6-month strategy arises for cumulative return, followed by 0.041% per day (*t*-statistic = 1.95) or 0.86% per month obtained by the R-ratio(91%,91%). Thus, the annualized differential return between cumulative return and the best alternative ratio is 5.04%. The Sharpe ratio achieves a momentum profit of 0.031% per day or 0.65% per month (*t*-statistic = 0.9103). The lowest monthly momentum return of 0.21% is obtained for the R-ratio(50%,95%). For the STARR ratios, the returns of winner minus loser portfolios fall within the range between 0.61% and 0.79% per month.

[Insert Table 1 here]

The R-ratio(91%,91%) or R-ratio(0.09, 0.09) obtains the best performance with

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[7] To obtain monthly returns, the daily returns are multiplied by 21 (assumption of 21 trading days per month).

[8] The application of *t*-statistics requires the i.i.d. normal assumption for its (asymptotic) validity. As alternative, we use the Kolmogorov-Smirnov statistics as a distance measure.

respect to realized average momentum profits when compared to other reward-risk ratio criteria. This ratio measures the reward and risk using the ETL measure at the 9% significance level of the right and left tail of distribution[9], respectively. The R-ratio with these parameters seems to capture well the distributional behavior of the data which is usually a component of risk due to heavy tails. The R-ratio(0.01, 0.01) that captures the risk of the extreme tail (i.e., covered by the measure of $ETL_{1\%}$) provides the second best performance among the R-ratios.

The STARR(95%) ratio that captures tail risk at the 95 percentile confidence level provides the best result on realized momentum profits among the STARR ratios (0.79% per month). Medium tail risk is measured by the STARR(75%) criterion which obtains the second best momentum profit among the STARR ratios. The STARR(50%) ratio covers the entire downside risk and obtains the average return that is slightly lower than that of the STARR(75%) ratio. These results suggest that increasing levels of risk aversion, as indicated by the STARR ratio parameter, seem to induce lower compensation in spread returns. The difference of realized momentum spreads between the largest and the lowest realized spread obtained by the STARR ratios is 0.19% per month, which translates to annualized return differential of 2.27%. The Sharpe ratio obtains approximately the half of the momentum profit of the cumulative return criterion.

Additional summary statistics (including measures of standard deviation, skewness and kurtosis for the momentum-sorted portfolio deciles) are reported in Table 1. The measure of skewness, $\hat{S}$, is calculated as

$$\hat{S} = \frac{1}{T}\sum_{t=1}^{T}\frac{\left(r_t - \bar{r}\right)^3}{\hat{\sigma}^3} \tag{5}$$

and the measure of kurtosis, $\hat{K}$, as

[9] The ETL measures the expected value of portfolio returns given that the VaR has been exceeded. When the ETL concept is symmetrically applied to the appropriate part of the portfolio returns, a measure of portfolio reward is obtained (see Biglova, Ortobelli, Rachev and Stoyanov, 2004).

$$\hat{K} = \frac{1}{T}\sum_{t=1}^{T}\frac{(r_t - \bar{r})^4}{3\hat{\sigma}^4} - 1 \qquad (6)$$

where T is the number of observations, $\bar{r}$ is the estimator of the first moment and $\hat{\sigma}$ is the estimator of the second moment. Under the assumptions of normality, $\hat{S}$ and $\hat{K}$ would have a mean zero asymptotic normal distribution with variances $6/T$ and $8/3T$, respectively.

For cumulative return, Sharpe ratio and STARR ratios, the volatility measure obtains the U-shape across the momentum deciles, consistent with Fama and French (1996) findings. For every criterion, the volatility of winner and loser portfolios is about the same magnitude with the slightly greater value for the loser portfolio. For the R-ratios there is no clear pattern of volatility measure across momentum deciles. The highest volatility estimates for winner and loser portfolios are obtained for cumulative return criterion with values of 0.01871 and 0.0171, respectively. The lowest estimates are obtained for the R-ratios with average approximate values of 0.013 and 0.012 for winner and loser portfolios, respectively. These results suggest that the strategy using cumulative return criterion is riskier than any other strategy when risk is measured based on volatility of extreme winner and loser portfolios.

The univariate statistics for the momentum deciles show a clear systematic pattern for the skewness measure across momentum portfolio deciles. For every criterion that we use on the dataset, the skewness of the loser portfolio is higher than that of the winner portfolio. $\hat{S}$ is large and positive for the portfolio decile 1 (loser) and decreases to a small positive or negative value $\hat{S}$ for the portfolio decile 10 (winner). This decrease in value is not strictly monotonic and the nature of the decrease of these values depends on the ratio criteria. Across all criteria, the average skewness for the loser portfolio is –0.0160 whereas the average skewness for the winner portfolio is 0.3108. The higher mean strategy is associated with lower skewness.

The obtained results for skewness differential between loser and winner portfolios are in line with the findings of Harvey and Siddique (2000), who examine a skewness pattern on a number of momentum datasets. For the cumulative return strategy on NYSE/AMEX and Nasdaq stocks with different ranking and holding periods, they conclude that the higher mean strategy is associated with lower skewness. Although we consider much shorter data-set than Harvey and Siddique, our conclusions are similar for an extended set of reward-risk stock selection criteria.

To illustrate the relation between the skewness and the momentum effect, Figure 1 plots the skewness measure across momentum portfolio deciles for cumulative return, Sharpe ratio, and STARR(95%) criterion.

[Insert Figure 1 here]

The $\hat{K}$ measures are largest for the loser portfolio decile, and are higher than those for the winner portfolio for every stock selection criterion.

In summary, the cumulative return criterion obtains the largest average monthly momentum profit among all criteria, and this strategy is riskier than other strategies measured on volatility of extreme winner and loser portfolios. When measured on volatility, the strategies based on the Sharpe ratio and the STARR ratios are less risky than the strategy on cumulative return criterion. The R-ratios obtain the lowest volatility of winner and loser portfolios among all the criteria. The evidence of a systematic skewness pattern across momentum portfolio deciles (with negative skewness differential between winner and loser portfolio) indicates that the higher mean strategy is associated with lower skewness. Given the dataset, the results suggest that the strategy criterion that obtains the higher average realized profits seems to accept higher volatility risk of extreme portfolios and negative skewness differential between winner and loser portfolios.

3.2 Final Wealth of Momentum Portfolios

For estimation of the final wealth of the momentum portfolio, we assume that the initial value of the winner and loser portfolios is equal to 1 and that the initial cumulative return, CR_0, is equal 0 at the beginning of the first holding period. We then obtain the total return of the winner and loser portfolios and their difference is the final wealth of the portfolio. Given continuously compounded returns, the cumulative return CR_n in each holding period, is given by

$$CR_k = CR_{k-1} + \sum_{i=1}^{T} \left(r_{iW} - r_{iL} \right) \quad \text{i = 1,…}T\text{; k} \geq 1 \qquad (7)$$

where r_{iW} and r_{iL} are the winner and loser portfolio daily returns at day *i*, *i* = 1,…,*T* and *T* is the number of days in the holding period *k, k* ≥ 1, respectively. The total cumulative return at the end of the entire observed period is the sum of the cumulative returns of every holding period.

Table 2 reports the result for the final wealth of the momentum portfolios at the end of the observation period (end of the last holding period) for every stock selection criterion. In general, the relative rankings of the final wealth values of different momentum strategies reflect those of average monthly profits from Table 1.

[Insert Table 2 here]

Final wealth for our 6-month/6-month strategy is positive for all reward-risk ratio criteria and cumulative return criterion. The highest value of the final wealth for winner-loser spread is obtained for cumulative return. Among the ratio criteria, the highest value of 3.6614 is obtained for STARR(95%), followed by the R-ratio(91%,91%) with a value of 3.3876. The lowest values for the final wealth of momentum portfolio are obtained for the the R-ratio(50%,99%) and R-ratio(50%,95%) with values of 0.9404 and 0.5864, respectively.

Figure 2 plots the cumulative realized profits (accumulated difference between winner and loser portfolio return over the whole period) to the 6-month/6-month strategy for the cumulative return, Sharpe ratio, and the R-ratio(91%,91%) criterion. The graph of the cumulative realized profits for the R-ratio(91%,91%) shows better performance than the graph for the Sharpe ratio given the value of the total realized return of the portfolio at the end of the observed period. The total realized return of the winner-loser portfolio for cumulative return criterion is higher than the total realized return for two observed ratios.

[Insert Figure 2 here]

3.3 Distributional Analysis of Momentum Portfolios using Estimates of Stable Paretian Distribution

It is a well-known fact supported by empirical evidence that financial asset returns do not follow normal distributions. Mandelbrot (1963) and Fama (1963, 1965) were the first to formally acknowledge this fact and build the framework of using stable distributions to model financial data. The excessively peaked, heavy-tailed and asymmetric nature of the return distribution made them reject the Gaussian hypothesis in favor of more general stable distributions, which can incorporate excess kurtosis, fat tails and skewness. Since this initial work in the 1960s, the stable distribution has been applied to modeling both the unconditional and conditional return distributions, as well as providing a theoretical framework of portfolio theory and market equilibrium models (Rachev and Mittnik, 2000).

The stable distribution is defined as the limiting distribution of sum of i.i.d. random variables. The class of all stable distributions can be described by four parameters $(\alpha, \beta, \mu, \sigma)$. The parameter α is the index of stability and must satisfy $0 < \alpha \leq 2$. When $\alpha = 2$, we obtain the Gaussian distribution. If $\alpha < 2$, moments of order α or higher do not exist and the tails of the distribution become heavier (i.e., the magnitude and frequency of outliers, relative to the Gaussian distribution, increases as α decreases). The parameter β determines skewness of the distribution and is within the range [-1, 1]. If $\beta = 0$, the distribution is symmetric. If $\beta > 0$, the distribution is skewed to the right and to the left if $\beta < 0$. Stable distributions therefore allow for skewed distributions when $\beta \neq 0$ and fat tails relative to the Gaussian distribution when $\alpha < 2$. The most important parameters are α and β since they identify two fundamental properties that are untypical of the normal distribution – heavy tails and asymmetry. The location is described by μ and σ is the scale parameter, which measures the dispersion of the distribution corresponding to the standard deviation in Gaussian distributions.

The α-stable, or S_α distribution in short, has generally no closed-form expression for its probability density function, but can, instead, be expressed by its characteristic function. The definition of a stable random variable X states that for $\alpha \in (0,2]$, $\beta \in [-1,1]$, and $\mu \in \Re$, X has the characteristic function of the following form

$$\varphi_X(t) = \begin{cases} \exp\left\{-\sigma^\alpha \, |t|^\alpha \left(1 - i\beta sign(t) \tan\frac{\pi\alpha}{2} + i\mu t\right)\right\} & if\ \alpha \neq 1 \\ \exp\{-\sigma\,|t|\,(1 + i\beta sign(t) \ln|t| + i\mu t)\} & if\ \alpha = 1 \end{cases} \quad (8)$$

Then the stable random variable X is denoted by $X \sim S_\alpha(\sigma, \beta, \mu)$. In particular, when both the skewness and location parameters β and μ are zero, X is said to be symmetric α-stable and denoted by $X \sim S\alpha S$. A stable distribution is therefore determined by the four key parameters: α (index of stability or tail index) determines density's kurtosis with $0 < \alpha \leq 2$, β determines density's skewness with $-1 \leq \beta \leq 1$, σ is a scale parameter (in the Gaussian case, $\alpha = 2$ and $2\sigma^2$ is the variance), and μ is a location parameter. If $1 < \alpha \leq 2$, then μ is the usual mean estimator.

We estimate the parameters of a stable distribution and approximate the stable density functions by applying a maximum likelihood estimation using Fast Fourier Transform (FFT). Details of the estimation procedure are given in Rachev and Mittnik (2000). The application of computationally demanding numerical approximation method in estimation of stable distribution is necessary, while closed-form expressions for its probability density function generally do not exist.

The momentum portfolio deciles obtained on different stock selection criteria are examined to understand how the decile returns relate to various estimates of the parameters of the stable distribution. The estimation results for the winner and loser portfolios and winner minus loser spread are reported in Table 3. Detailed results for every criterion and momentum decile are provided in Table 4.

[Insert Table 3 here]

The estimate of the index of stability α has the lowest value for the loser portfolio for every criterion. The α value for winner portfolio is higher than that of loser portfolio for every criterion and has the second lowest value for some criteria. The values of the tail index for the loser deciles are in the range [1.567, 1.568] and for the winner deciles in the range [1.767, 1.768]. The lowest tail index of 1.5670 for the loser portfolio is obtained for the cumulative return criterion and the largest for the R-ratio(99%,99%). For the cumulative return criterion, tail index estimates of deciles P1 to P4 are lower than those of any other criterion. The tail index estimate for the loser portfolio of the Sharpe ratio and the STARR criteria is slightly larger than that of the cumulative return. The highest tail index estimates of winner and loser portfolios are obtained for the R-ratios. The results suggest that the higher mean strategy is associated with higher tail index. Since the R-ratios obtain the highest α estimates for the winner and loser portfolios, they accept less heavy tail distributions and reduce tail risk. On the contrary, cumulative return criterion accepts the heaviest tail distribution of the loser portfolio with substantial tail risk. The results indicate that the strategies using cumulative return and alternative return criteria require acceptance of heavy tail distributions (with tail index below two), implying heavy tail risk when constructing zero-investment portfolios.

For the cumulative return, Sharpe ratio, every STARR ratio and the R-ratio(91%,91%) criterion, the estimate of skewness (β) of the loser portfolio is higher than that of the winner portfolio, consistent with the results under the normal distribution assumption. Hence, strategies using all ranking criteria except some R-ratios require acceptance of negative skewness. Thus, the higher mean strategy is associated with lower skewness for cumulative return, Sharpe ratio and STARR ratio.

The estimates of the scale parameter σ for cumulative return, Sharpe ratio, and STARR ratios are generally the highest for the winner and loser deciles, with the value for the winner decile higher than that for the loser decile. These observations for the stated criteria are similar to the results for the standard deviation estimates across momentum portfolio deciles. For the R-ratios, the estimates of the scale parameter σ for the loser decile is lower than estimates of the middle deciles and the winner decile. For the R-ratio(91%,91%), R-ratio(50%,99%), and R-ratio(50%,95%), the scale estimate from loser to winner portfolio is almost monotonically increasing.

The results suggest that in the 6/6 momentum strategy on the observed data set, constructing zero-investment portfolio requires acceptance of heavy tail distributions (with tail index less than two) for every criterion. This suggests that winner and loser portfolio returns imply significant risk component due to heavy tails when compared to middle deciles. The effect of using alternative STARR ratios and R-ratios compared to the cumulative return criterion is that they increase the tail index estimate of the loser portfolio which implies reduction of the tail risk in the loser portfolio. The effect of using the R-ratios compared to the cumulative return benchmark is that they increase the tail index of both the loser and winner portfolios, where the impact on the loser portfolio is more pronounced. This suggests that the R-ratios accept less heavy-tailed distributions and reduce tail risk. The highest tail index estimates for winner and loser portfolios are obtained for the R-ratio(99%,99%) and R-ratio(95%,95%), implying that they are the most effective in reducíng risk due to heavy tails.

[Insert Table 4 here]

Figure 3 shows the estimate of the tail index across the momentum deciles for the cumulative return, Sharpe ratio, and the R-ratio(91%,91%) criterion. It is evident that the extreme momentum deciles (winner and loser) generally have lower tail index estimates than the middle deciles for the considered criteria.

[Insert Figure 3 here]

For the return distribution of every momentum decile, the estimated values of the index of stability α are below 2 and there is obvious asymmetry ($\beta \neq 0$). These facts strongly suggest that the Gaussian assumption is not a proper theoretical distribution model for describing the momentum portfolio return distribution. In addition, the extreme momentum portfolios (winner and loser) show unique characteristics regarding the estimates of the tail index and skewness.

To further investigate the non-normality of the momentum decile returns, we assess whether the Gaussian distribution hypothesis holds for momentum decile returns and compare it to the stable Paretian distribution hypothesis. We assume that momentum decile daily return observations are independent and identically distributed (i.i.d.) and therefore the test considers an unconditional, homoscedastic distribution model. For the tests of normality, we employ two goodness-of-fit tests which are based on the whole empirical distribution, the Kolmogorov distance (KD) statistic and the Anderson-Darling (AD) statistic. For the i.i.d. model, we compare the goodness-of-fit in the case of the Gaussian hypothesis and the more general stable Paretian hypothesis.

The KD-statistic is computed according to

$$KD = \sup_{x \in R} | F_S(x) - \hat{F}(x) | \tag{9}$$

where $F_S(x)$ is the empirical sample distribution and $\hat{F}(x)$ is the cumulative density function (cdf) of the estimated parametric density. This statistic emphasizes the deviations around the median of the fitted distribution. It is a robust measure in the sense that it focuses only on the maximum deviation between the sample and fitted distributions.

The AD-statistic is computed as follows

$$AD = \sup_{x \in R} \frac{| F_S(x) - \hat{F}(x) |}{\sqrt{\hat{F}(x)\left(1 - \hat{F}(x)\right)}} \tag{10}$$

The AD statistic accentuates the discrepancies in the tails. Thus, we compare the empirical sample distribution of momentum decile residuals with either an estimated Gaussian or a non-Gaussian distribution.

The last two columns of Table 3 show the KD-statistic values for the winner and loser portfolio for all ranking criteria. The KD statistics indicate that the Gaussian hypothesis is rejected at the 5% significance level for the winner and loser portfolio as well as momentum spread of every ranking criterion. In contrast, the stable-Paretian hypothesis is not rejected for the winner and loser portfolios and momentum spread of any criterion. Detailed results of the goodness-of-fit statistics for the fitted unconditional distributions of the momentum deciles are provided in Table 5. The KD statistics reject the Gaussian hypothesis for all momentum deciles and criteria at the 95% confidence level. However, the KD statistics show that we cannot reject the stable distribution for all deciles and criteria except one (i.e., decile P5 for the R-ratio(50%,95%)). The better fit of the stable distribution assumption over the Gaussian distribution assumption for momentum decile returns is confirmed by analyzing the values of KD and AD statistics. For every momentum decile and criterion, the KD statistic in the stable Paretian case is below the KD statistic in the Gaussian case. The same conclusion applies to the AD statistic. The substantial difference between the AD statistic computed for the stable Paretian model relative to the Gaussian model shows that in reality extreme events have larger probability than predicted by the Gaussian distribution, especially in the winner and loser deciles. The obtained results on the KD and AD tests suggest that the stable Paretian distribution hypothesis is superior to the normal distribution hypothesis in describing the unconditional distribution of momentum portfolio returns. This implies that the stable Paretian distribution hypothesis better explains the tails and the central part of the return distribution.

Figures 4 and 5 plot the density functions for the winner and loser portfolios of the cumulative return criterion. Density functions of the winner and loser portfolio display features of asymmetry and heavy tails with a higher peak than the normal distribution. The heavy tail effect in the loser portfolio is more pronounced than for the winner portfolio, with the distribution exhibiting fatter tails and more peakedness associated with lower value of index of stability. The markedly better fit of the distributions under stable Paretian hypothesis relative to the Gaussian hypothesis is evident, suggesting a much better ability of the stable Paretian model to explain extreme events.

[Insert Figure 4 here]

[Insert Figure 5 here]

Figures 6 and 7 plot the density functions for the winner and loser portfolios of the R-ratio(91%,91%) criterion. Density functions of extreme portfolios display features of asymmetry and heavy tails with a higher peak than the normal distribution. Contrary to the cumulative return criterion, the peakedness of both distributions is similar. The better fit of the stable Paretian model relative to the Gaussian model is also evident.

[Insert Figure 6 here]

[Insert Figure 7 here]

3.4 Performance Evaluation of Risk-Return Ratio Criteria Based on Independent Performance Measure

We apply an independent risk-adjusted performance measure[10] to evaluate ratio criteria on an appropriate measure of risk in a uniform manner. To that purpose, we introduce a risk-adjusted independent performance measure, $E(X_t)/CVaR_{99\%}(X_t)$, in the form of the $STARR_{99\%}$ ratio, where X_t is the sequence of the daily spreads (difference between winner and loser portfolio returns over the observed period). The stock ranking criterion that obtains the best risk-adjusted performance is the one that attains the highest value of the independent performance measure.

Different independent performance measures can be created by changing the significance level of the ETL measure that considers downside risk, reflecting different risk-return profile objectives and levels of risk-aversion of an investor based on which the various criteria are evaluated. The best performance of certain criterion on some risk-adjusted measure amounts to the best out-of-sample performance in the holding periods according to a chosen risk-return profile and risk-aversion preference.

The results of the evaluation of the reward-risk criteria based on the cumulative realized profit, Sharpe measure (i.e., we use this term in the context of strategy evaluation), and independent performance measure are shown in Table 6. For the 6-month/6-month strategy, the best performance value of 0.0218 on the independent performance measure is obtained for the R-ratio(99%,99%). The next two best values of 0.0187 and 0.0155 are obtained by the R-ratio(91%,91%) and R-ratio(95%,95%), respectively. The cumulative return criterion obtains the fourth highest value of 0.01398

[10] Unlike other studies which apply one-factor or multi-factor models as the performance benchmark, we give preference to direct evaluation and comparison of performance on final wealth and risk-adjusted performance measure using daily data. The portfolio alpha's values from the factor models can be considerably influenced by the overly simplistic structure of the model with much of the idiosyncratic risk left unexplained.

and the Sharpe ratio obtains the fourth lowest value of 0.0139. These results imply that the Sharpe ratio as the risk-adjusted criterion for a given set of data is not providing an optimal risk-adjusted performance. The risk adjusted performance of the best R-ratio is approximately 150% times better than that of the cumulative return criterion and almost three times better than that of the Sharpe ratio.

To summarize, although the cumulative return criterion obtains the largest cumulative profits, the alternative R-ratios obtain the best risk-adjusted performance based on the applied independent performance measure. If we use the independent risk-adjusted performance measure in the form of the STARR ratio with different significance levels of the parameter, the results on the ranking of the criteria may change, reflecting different risk-return objectives and levels of risk-aversion of the investor.

[Insert Table 6 here]

5. Conclusions

In this study, we apply the alternative reward-risk criteria to momentum portfolio construction. The alternative ratio criteria that account for a risk-return profile of the individual stocks are based on the coherent risk measure of the expected shortfall and are not restricted to the normal return distribution assumption. The key distinctive properties of alternative ratios is that they only assume finite mean of the individual stock return distribution and can model different levels of risk aversion using different parameters for significance level of the ETL measure that considers different parts of downside risk. Additionally, reward-risk ratio criteria values are computed using daily data which enable them to better capture the distributional properties of stock returns and their risk component at the tail part of distribution. Alternative ratios drive balanced risk-return performance according to captured risk-return profiles of observed stocks in a sample. For examined 6-month/6-month strategy, although the cumulative return criterion provides the highest realized annualized return of 15.36%, the alternative R-ratio provides a high annualized return of 10.32% and much better risk-adjusted performance than a simple cumulative return and the traditional Sharpe ratio criterion.

Distributional analysis of the momentum deciles within a framework of general stable distributions indicates that the stable Paretian distribution hypothesis provides a much better fit to momentum portfolio returns. Moreover, extreme winner and loser decile portfolios have unique characteristics with regards to stable parameter estimates of their returns. We observe a systematic pattern of index of stability for extreme winner and loser deciles that have generally lower tail index than that of middle deciles. It is not surprising that those assets should also exhibit the highest tail-volatility of the return distributions. This suggests that winner and loser portfolio returns imply a substantial risk component due to heavy tails when compared to other deciles. The implication is that momentum strategies require acceptance of heavy-tail distributions (with a tail index below two). As a consequence, an investor that considers only the cumulative return criterion for momentum strategy needs to accept the heavier tail distributions and greater heavy tail risk than the investor that pursue strategies based on alternative reward-risk criteria. Alternative reward-risk strategies exhibit better risk-adjusted returns with lower tail risk. Furthermore, strategies using cumulative return, Sharpe ratio and STARR ratio criteria require acceptance of the negative skewness.

The results for risk-adjusted performance of different reward-risk ratios and cumulative return benchmark using the $STARR_{99\%}$ ratio for daily spreads confirm that the alternative R-ratio and the STARR ratios capture the distributional behavior considerably better than the classical mean-variance model underlying the Sharpe ratio. The Sharpe ratio criterion underperforms based on the cumulative profit and independent performance measure. The reason behind the better risk-adjusted performance of the alternative ratios lies in the compliance with the coherent risk measure's ability to capture distributional features of data including the component of risk due to heavy tails, and the property of parameters in the R-ratio to adjust for the upside reward and downside risk simultaneously.

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Footnotes

1. The observation by Mandelbrot (1963) and Fama (1963, 1965) of excess kurtosis in empirical financial return processes led them to reject the normal assumption and propose non-Gaussian stable processes as a statistical model for asset returns. Non-Gaussian stable distributions are commonly referred to as "stable Paretian" distribution due to the fact that the tails of the non-Gaussian stable density have Pareto power-type decay.
2. In the mean-variance space, winners are positioned on the upper part of the efficient mean-variance frontier, so that by construction they entail the highest risk Losers simply correspond to the winners multiplied by -1. In the observed universe of assets bounded by mean-variance frontier, there may be riskier assets with higher variance (measured by standard deviation) than those of selected winners (or losers) but they will not qualify as winner or loser based on cumulative return criterion. Similar to return ranking criterion, we might think of applying a variance ranking criterion. However, the stocks with the greatest variance do not necessarily need to be the stocks with the greatest return at the same time and would not be on the efficient frontier.
3. This choice can be further complicated by considering which risk-adjusted criteria are more useful for an investor rather than another.
4. There are some notational inconsistencies in the current literature on expected shortfall. Some authors distinguish between expected tail loss and CVaR, so that only in the case of continuous random variable the definition of expected shortfall coincides with that of CVaR.
5. Expected shortfall conveys the information about the expected size of a loss exceeding VaR. For example, suppose that a portfolio's risk is calculated through the simulation. For 1,000 simulations and α= 0.95, the portfolio's VaR would be the smallest of the 50 largest losses. The corresponding expected shortfall would be then estimated by the numerical average of these 50 largest losses.

6. R-ratio stands for Rachev ratio and was introduced in the context of risk-reward alternative performance measures and risk estimation in portfolio theory (Biglova, Ortobelli, Rachev, and Stoyanov, 2004).

7. To obtain monthly returns, the daily returns are multiplied by 21 (assumption of 21 trading days per month)

8. The application of *t*-statistics requires the i.i.d. normal assumption for its (asymptotic) validity. As alternative, we use the Kolmogorov-Smirnov statistics as a distance measure.

9. The ETL measures the expected value of portfolio returns given that the VaR has been exceeded. When the ETL concept is symmetrically applied to the appropriate part of the portfolio returns, a measure of portfolio reward is obtained (see Biglova, Ortobelli, Rachev and Stoyanov, 2004).

10. Unlike other studies which apply one-factor or multi-factor models as performance benchmark, we give preference to direct evaluation and comparison of performance on final wealth and risk-adjusted performance measure using daily data. The portfolio alpha's values from the factor models can be considerably influenced by the overly simplistic structure of the model with much of the idiosyncratic risk left unexplained.

Figure Captions

Figure 1. Estimate of skewness for momentum decile returns of a 6-month/6-month momentum strategy for cumulative return, Sharpe ratio and STARR(95%) criterion

Figure 2: Cumulative realized momentum profits for cumulative return, Sharpe ratio and R-ratio(91%,91%) criterion of the 6-month/6-month strategy.

Figure 3: Estimates of the tail index for momentum decile returns of the 6-month/6-month momentum strategy for cumulative return, Sharpe ratio and R-ratio(91%,91%)

Figure 4. Density plot for the winner portfolio of the momentum dataset obtained by cumulative return criterion.

Figure 5. Density plot for the loser portfolio of the momentum dataset obtained by cumulative return criterion.

Figure 6. Density plot for the winner portfolio of the momentum dataset obtained by R-ratio(91%,91%) return criterion.

Figure 7. Density plot for the loser portfolio of the momentum dataset obtained by R-ratio(91%,91%) return criterion.

Table 1. Momentum portfolio returns and summary statistics

Reward-risk ratio	Portfolio	Summary Statistics			
		Mean	Std. Dev.	Skewness	Kurtosis
Cumulative return (Benchmark)	Winner	0.00143	0.0187	-0.0163	2.9218
	Loser	0.00082	0.0170	0.4814	5.5751
	Winner-Loser	0.00061	0.0170	-0.0559	6.5882
Sharpe Ratio	Winner	0.00110	0.0152	0.0177	4.6177
	Loser	0.00079	0.0148	0.5146	5.9533
	Winner - Loser	0.00031	0.0147	-0.7383	10.0386
STARR (99%)	Winner	0.00108	0.0148	-0.0134	3.8088
	Loser	0.00079	0.0155	0.5780	5.5742
	Winner-Loser	0.00029	0.0146	-0.7550	8.8461
STARR (95%)	Winner	0.00117	0.0152	-0.0206	4.2520
	Loser	0.00079	0.0152	0.5300	5.2106
	Winner-Loser	0.00037	0.0147	-0.6688	8.5333
STARR (90%)	Winner	0.00112	0.0152	-0.0096	4.0685
	Loser	0.00079	0.0151	0.5095	5.7897
	Winner-Loser	0.00033	0.0146	-0.6826	8.5087
STARR (75%)	Winner	0.00112	0.0150	0.0030	4.1790
	Loser	0.00076	0.0148	0.5085	5.9717
	Winner-Loser	0.00036	0.0844	-0.7750	9.6975
STARR (50%)	Winner	0.00113	0.0149	0.0023	4.5457
	Loser	0.00078	0.0146	0.5153	6.1469
	Winner-Loser	0.00035	0.0144	-0.7534	10.2474
R-ratio (99%,99%)	Winner	0.00102	0.0131	-0.0285	2.6163
	Loser	0.00066	0.0116	-0.0073	2.3276
	Winner-Loser	0.00036	0.0073	-0.1968	3.1379
R-ratio (95%,95%)	Winner	0.00102	0.0134	-0.0616	2.9597
	Loser	0.00070	0.0118	0.0268	3.1761
	Winner-Loser	0.00031	0.0085	-0.2281	5.1429
R-ratio (91%,91%)	Winner	0.00108	0.0135	-0.1261	3.1018
	Loser	0.00067	0.0117	0.0375	3.3995
	Winner-Loser	0.00041	0.0091	-0.4209	5.4828
R-ratio (50%,99%)	Winner	0.00091	0.0144	0.0613	2.7349
	Loser	0.00076	0.0123	0.0222	3.7835
	Winner-Loser	0.00015	0.0097	-0.0791	5.6342
R-ratio (50%,95%)	Winner	0.00096	0.0145	-0.0002	2.8884
	Loser	0.00086	0.0122	0.0126	4.6842
	Winner-Loser	0.00010	0.0108	-0.1949	4.6484

For every 6 month strategy, the sample of 517 stocks in the S&P 500 universe is ranked over the period January 1996 to December 2003 into 10 equally weighted portfolios and returns evaluated over holding period of 6 months. The above statistics are based on daily portfolio returns. The table reports average return, standard deviation, skewness and kurtosis for the loser and winner portfolio. "Loser" is the equally weighted portfolio of 10% of the stocks with the lowest values of the criteria over the past 6-months, and "Winner" comprises the 10% of the stocks with the highest values of the criteria over the past 6-months.

Table 2. Final Wealth of Momentum Portfolios

Reward-risk Ratio	Portfolio	Final Wealth
Cumulative return (Benchmark)	Loser	3.5658
	Winner	10.5592
	Winner-Loser	6.9934
Sharpe Ratio	Loser	3.6141
	Winner	6.4059
	Winner - Loser	2.7917
STARR (99%)	Loser	3.5306
	Winner	6.1856
	Winner-Loser	2.6550
STARR (95%)	Loser	3.6038
	Winner	7.2652
	Winner-Loser	3.6614
STARR (90%)	Loser	3.5744
	Winner	6.6241
	Winner-Loser	3.0497
STARR (75%)	Loser	3.4283
	Winner	6.7135
	Winner-Loser	3.2852
STARR (50%)	Loser	3.5729
	Winner	6.8325
	Winner-Loser	3.2596
R-ratio (99%,99%)	Loser	3.0539
	Winner	5.844946
	Winner-Loser	2.7913
R-ratio (95%,95%)	Loser	3.3021
	Winner	5.7529
	Winner-Loser	2.4508
R-ratio (91%,91%)	Loser	3.1119
	Winner	6.4995
	Winner-Loser	3.3875
R-ratio (50%,99%)	Loser	3.6155
	Winner	4.5559
	Winner-Loser	0.9404

R-ratio (50%,95%)	Loser	4.4227
	Winner	5.0091
	Winner-Loser	0.5864

This table reports the final wealth for momentum portfolios at the end of observed period for different reward-risk selection criteria. The final wealth of winner and loser portfolios is the total realized return on these portfolios at the end of observed period respectively. Loser (P1) is the equally weighted portfolio of 10% of the stocks with the lowest values of criteria over the past *6*-months, and winner (P10) comprises the stocks with the highest values of criteria over the past *6*-months. Annualized returns for the specific strategy and risk-return ratio are given in parentheses. The sample includes a total of 500 stocks during the period of January 1996 to December 2003.

Table 3. Estimation of stable parameters for momentum portfolio returns and goodness-of-fit statistics

Reward-risk ratio	Portfolio	Stable parameter estimates					
		α	β	σ	µ	KD normal	KD stable
Cumulative return (Benchmark)	Winner	1.75007	-0.07045	0.01116	0.00143	0.0467*	0.0226
	Loser	1.56704	0.03715	0.00878	0.00082	0.0774*	0.0160
	Winner-Loser	1.47653	-0.04171	0.00790	0.00077	0.0929*	0.0269
Sharpe Ratio	Winner	1.65848	-0.09464	0.00836	0.00110	0.0695*	0.0204
	Loser	1.59524	0.00859	0.00775	0.00079	0.0852*	0.0177
	Winner - Loser	1.37676	-0.00719	0.00596	0.00057	0.1295*	0.0180
STARR(99%)	Winner	1.70630	-0.08165	0.00848	0.00108	0.0556*	0.0259
	Loser	1.57861	0.02845	0.00809	0.00079	0.0708*	0.0210
	Winner-Loser	1.39529	-3.7567E-05	0.00634	0.00058	0.1186*	0.0154
STARR(95%)	Winner	1.68552	-0.06897	0,00856	0.00117	0.0605*	0.0145
	Loser	1.58688	-0.00556	0.00797	0.00079	0.0754*	0.0176
	Winner-Loser	1.37519	0.01486	0.00619	0.00075	0.1232*	0.0182
STARR(90%)	Winner	1.66926	-0.07408	0.00844	0.00112	0.0680*	0.0167
	Loser	1.58386	-0.00448	0.00779	0.00079	0.0807*	0.0200
	Winner-Loser	1.35305	0.02935	0.00592	0.00080	0.1345*	0.0157
STARR(75%)	Winner	1.66501	-0.08464	0.00829	0.00112	0.0681*	0.0239
	Loser	1.59155	-0.00101	0.00776	0.00076	0.0767*	0.0169
	Winner-Loser	1.36638	0.01084	0.00580	0.00071	0.1298*	0.0174
STARR(50%)	Winner	1.65393	-0.09623	0.00816	0.00113	0.0646*	0.0116
	Loser	1.59691	0.00476	0.00765	0.00078	0.0711*	0.0201
	Winner-Loser	1.37468	0.00650	0.00578	0.00069	0.1296*	0.0248
R-ratio (99%,99%)	Winner	1.77798	0.03522	0.00804	0.00102	0.0454*	0.0223
	Loser	1.77270	-0.06577	0.00713	0.00066	0.0536*	0.0189
	Winner-Loser	1.73249	-0.09520	0.00427	0.00032	0.0507*	0.0208
R-ratio (95%,95%)	Winner	1.78118	-0.04382	0.00814	0.00102	0.0503*	0.0219
	Loser	1.75262	-0.07338	0.00705	0.00070	0.0543*	0.0225
	Winner-Loser	1.64026	-0.05710	0.00453	0.00028	0.0708*	0.0123
R-ratio (91%,91%)	Winner	1.78587	-0.08033	0.00822	0.00108	0.0496*	0.0224
	Loser	1.71969	-0.04877	0.00687	0.00067	0.0600*	0.0237
	Winner-Loser	1.61345	0.01532	0.00474	0.00054	0.0839*	0.0175
R-ratio (50%,99%)	Winner	1.79783	0.07168	0.00886	0.00091	0.0506*	0.0256
	Loser	1.69111	-0.06882	0.00701	0.00076	0.0566*	0.0219
	Winner-Loser	1.62541	0.01079	0.00510	0.00019	0.0736*	0.0222
R-ratio (50%,95%)	Winner	1.78654	0.04082	0.00884	0.00096	0.0459*	0.0205
	Loser	1.69439	-0.11894	0.00687	0.00086	0.0627*	0.0266
	Winner-Loser	1.56454	-0.02473	0.00553	0.00012	0.0762*	0.0236

For every 6 month strategy, the sample of 517 stocks in the S&P 500 universe is ranked over the period January 1996 to December 2003 into 10 equally weighted portfolios and returns are evaluated over holding period of 6 months. The estimates are based on daily portfolio returns. The table reports location parameter μ, index of stability α, skewness parameter β and the scaling parameter σ for the winner and loser portfolio and winner minus loser spread. For $\alpha > 1$, the location parameter μ is the usual mean estimator. Loser is the equally weighted portfolio of 10% of the stocks with the lowest values of the criteria over the past 6-months, and winner comprises the 10% of the stocks with the highest values of the criteria over the past 6-months. The values of the KD test for normal Gaussian hypothesis are given in parentheses (* denotes significance at the 5% level, critical value of 0.0338 for the KD test)

Table 4. Momentum Portfolio Returns and Estimates of Parameters of Stable Distribution

Reward-risk Ratio	Parameter estimate	Momentum deciles									
		P1 (Losers)	P2	P3	P4	P5	P6	P7	P8	P9	P10 (Winners)
Cumulative return (Benchmark)	μ	0.00082	0.00063	0.00054	0.00061	0.00067	0.00064	0.00073	0.00070	0.00089	0.00143
	α	1.56704	1.63614	1.71760	1.72691	1.79380	1.80571	1.79596	1.83524	1.8683	1.7500
	β	0.03715	-0.0133	-0.0118	-0.0809	0.0082	-0.0783	-0.0889	-0.0308	0.0944	-0.0704
	σ	0.00878	0.00715	0.00682	0.00639	0.00675	0.00664	0.00693	0.00720	0.0084	0.0111
Sharpe Ratio	μ	0.00079	0.00066	0.00061	0.00069	0.00065	0.00082	0.00068	0.00080	0.00083	0.00110
	α	1.5952	1.6733	1.7555	1.7317	1.7769	1.8471	1.8419	1.8407	1.8054	1.6584
	β	0.0086	-0.0046	-0.0220	-0.0649	0.0774	-0.1216	0.0613	-0.0201	-0.0182	-0.0946
	σ	0.0077	0,0076	0.0074	0.0071	0.0073	0.0072	0.0076	0.0073	0.0078	0.0083
STARR (99%)	μ	0.00079	0.00064	0.00061	0.00062	0.00084	0.00074	0.00067	0.00073	0.00087	0.00108
	α	1.5786	1.6600	1.7256	1.7615	1.7949	1.8470	1.8562	1.8263	1.8257	1.7063
	β	0.0284	-0.0276	-0.0438	-0.0702	0.0498	-0.0817	-0.0536	0.0484	0.0120	-0.0816
	σ	0.0080	0.0074	0.0073	0.0070	0.0073	0.0073	0.0076	0.0073	0.0080	0.0084
STARR (95%)	μ	0.00079	0.00065	0.00062	0.00060	0.00075	0.00081	0.00068	0.00080	0.00078	0.00117
	α	1.5868	1.6648	1.7438	1.7428	1.7895	1.8360	1.8498	1.8564	1.7861	1.6855
	β	-0.0055	0.0100	-0.0572	-0.0576	0.0397	-0.0073	-0.0432	0.0043	0.0277	-0.0689
	σ	0.0079	0.0075	0.0073	0.0069	0.0073	0.0072	0.0075	0.0075	0.0077	0.0085
STARR (90%)	μ	0.00079	0.00069	0.00059	0.00065	0.00067	0.00085	0.00067	0.00080	0.00082	0.00112
	α	1.5838	1.6793	1.7574	1.7452	1.7905	1.8532	1.8532	1.8516	1.7973	1.6692
	β	-0.0044	0.0169	-0.0183	-0.0478	0.0721	-0.0778	-0.0107	-0.0188	-0.0071	-0.0740
	σ	0.0078	0.0076	0.0074	0.0069	0.0074	0.0072	0.0076	0.0074	0.0077	0.0084
STARR (75%)	μ	0.00076	0.00068	0.00061	0.00063	0.00070	0.00080	0.00069	0.00078	0.00086	0.00112
	α	1.5915	1.6600	1.7509	1.7621	1.7871	1.8445	1.8382	1.8388	1.8118	1.6650
	β	-0.0010	-0.0079	-0.0068	-0.0279	0.0681	-0.0789	-0.0041	-0.0129	-0.0061	-0.0846
	σ	0.0077	0.0077	0.0073	0.0072	0.0074	0.0072	0.0076	0.0074	0.0079	0.0082
STARR (50%)	μ	0.00078	0.00068	0.00061	0.00064	0.00068	0.00081	0.00068	0.00078	0.00085	0.00113
	α	1.5969	1.6611	1.7554	1.7410	1.7846	1.8408	1.8445	1.8388	1.8125	1.6539
	β	0.0047	-0.0123	0.0031	-0.0424	0.0970	-0.1204	0.0362	-0.0164	-0.0192	-0.0962
	σ	0.0076	0.0077	0.0073	0.0072	0.0074	0.0072	0.0076	0.0074	0.0079	0.0081

Table 4. Cont.

Reward-risk Ratio	Parameter Estimate	Momentum deciles									
		P1 (Losers)	P2	P3	P4	P5	P6	P7	P8	P9	P10 (Winners)
R-ratio (99%,99%)	μ	0.00066	0.00067	0.00066	0.00068	0.00075	0.00076	0.00084	0.00081	0.00080	0.00102
	α	1.7727	1.7526	1.7927	1.8015	1.7909	1.8082	1.8152	1.8028	1.7285	1.7779
	β	-0.0657	-0.1155	-0.0659	-0.1098	0.0011	0.0375	0.1399	0.0722	0.0027	0.0352
	σ	0.0071	0.0072	0.0070	0.0071	0.0073	0.0074	0.0079	0.0077	0.0076	0.0080
R-ratio (95%,95%)	μ	0.00070	0.00055	0.00068	0.00072	0.00075	0.00073	0.00077	0.00077	0.00089	0.00102
	α	1.7526	1.7539	1.8136	1.7333	1.8190	1.8073	1.7754	1.8325	1.7624	1.7811
	β	-0.0733	-0.0690	-0.0497	-0.1063	0.0125	0.0908	0.0837	0.1161	0.0028	-0.0438
	σ	0.0070	0.0069	0.0072	0.0070	0.0074	0.0075	0.0079	0.0075	0.0080	0.0081
R-ratio (91%,91%)	μ	0.00067	0.00061	0.00070	0.00071	0.00070	0.00069	0.00077	0.00086	0.00083	0.00108
	α	1.7196	1.7652	1.7981	1.7679	1.7867	1.7894	1.8202	1.7799	1.8128	1.7858
	β	-0.0487	-0.0940	-0.1246	-0.0488	0.0182	0.0580	0.1854	0.0705	0.0325	-0.0803
	σ	0.0068	0.0072	0.0072	0.0071	0.0072	0.0073	0.0079	0.0078	0.0082	0.0082
R-ratio (50%,99%)	μ	0.00076	0.00071	0.00078	0.00077	0.00070	0.00074	0.00079	0.00063	0.00083	0.00091
	α	1.6911	1.7994	1.7097	1.8352	1.7553	1.7889	1.7862	1.8147	1.7990	1.7978
	β	-0.0688	-0.0688	-0.1138	0.0186	-0.1043	0.0621	0.0053	0.0682	0.0523	0.0716
	σ	0.0070	0.0071	0.0072	0.0072	0.0072	0.0075	0.0073	0.0076	0.0079	0.0088
R-ratio (50%,95%)	μ	0.00086	0.00073	0.00067	0.00070	0.00068	0.00071	0.00074	0.00081	0.00078	0.00096
	α	1.6943	1.7218	1.7352	1.7714	1.7694	1.8071	1.8376	1.8013	1.7956	1.7865
	β	-0.1189	-0.0815	-0.0397	-0.0780	-0.0315	0.0344	0.1321	-0.0334	-0.0196	0.0408
	σ	0.0068	0.0071	0.0070	0.0072	0.0074	0.0075	0.0075	0.0075	0.0084	0.0088

For every 6 month strategy, the sample of 517 stocks in the S&P universe is ranked over the period January 1996 to December 2003 into 10 equally weighted portfolios and returns are evaluated over holding period of 6 months. The estimates are based on daily portfolio returns. The table reports the location parameter μ, index of stability α, skewness parameter β and the scaling parameter σ for the momentum deciles. For $\alpha > 1$, as in the case here, the location parameter μ is the usual mean estimator. Loser (P1) is the equally weighted portfolio of 10% of the stocks with the lowest value of the criteria over the past 6-months, and winner (P10) is the equally weighted portfolio of 10% of the stocks with the highest value of the criteria over the past 6-months.

Table 5. Goodness-of-fit Statistics for Distribution of Momentum Portfolio Returns

Reward-risk ratio	Goodness-of-Fit statistic	Momentum deciles									
		P1 (Losers)	P2	P3	P4	P5	P6	P7	P8	P9	P10 (Winn-ers)
Cumulative return (Benchmark)	KD normal	0.0774	0.0745	0.0572	0.0588	0.0514	0.0414	0.0570	0.0377	0.0370	0.0467
	KD stable	0.0160	0.0297	0.0210	0.0182	0.0194	0.0183	0.0274	0.0168	0.0181	0.0226
	AD normal	0.2548	0.4353	0.5726	0.3607	0.3607	0.4137	0.3078	0.3607	0.3078	0.4137
	AD stable	0.0798	0.0857	0.0715	0.0604	0.0455	0.0458	0.0572	0.0458	0.0520	0.0510
Sharpe Ratio	KD normal	0.0852	0.0619	0.0578	0.0488	0.0561	0.0425	0.0493	0.0356	0.0492	0.0695
	KD stable	0.0177	0.0256	0.0221	0.0267	0.0252	0.0160	0.0232	0.0160	0.0258	0.0204
	AD normal	0.3607	0.3607	0.6785	0.3773	0.4137	0.3607	0.3078	0.2105	0.3607	0.5196
	AD stable	0.0695	0.0715	0.0600	0.0678	0.0584	0.0493	0.0526	0.0476	0.0601	0.0584
STARR (99%)	KD normal	0.0708	0.0596	0.0536	0.0526	0.0422	0.0482	0.0379	0.0530	0.0443	0.0556
	KD stable	0.0210	0.0176	0.0223	0.0273	0.0160	0.0226	0,0170	0.0312	0.0190	0.0259
	AD normal	0.4137	0.3607	0.4137	0.4667	0.2579	0.3607	0.4137	0.2548	0.3607	0.5102
	AD stable	0.0908	0.0633	0.0684	0.0614	0.0507	0.0499	0.0505	0.0742	0.0574	0.0540
STARR (95%)	KD normal	0.0754	0.0596	0.0552	0.0590	0.0520	0.0411	0.0350	0.0339	0.0500	0.0605
	KD stable	0.0176	0.0249	0.0267	0.0264	0.0240	0.0225	0.0198	0.0211	0.0259	0.0145
	AD normal	0.3607	0.4137	0.4727	0.2579	0.3607	0.3607	0.4137	0.3078	0.4667	0.3607
	AD stable	0.0756	0.0609	0.0592	0.0641	0.0549	0.0496	0.0436	0.0564	0.0563	0.0542
STARR (90%)	KD normal	0.0807	0.0610	0.0468	0.0472	0.0475	0.0335	0.0451	0.0399	0.0409	0.0680
	KD stable	0.0200	0.0249	0.0249	0.0209	0.0261	0.0171	0.0238	0.0257	0.0224	0.0167
	AD normal	0.3607	0.4667	0.6785	0.6785	0.3607	0.4137	0.2548	0.2548	0.2548	0.5196
	AD stable	0.0722	0.0726	0.0712	0.0549	0.0555	0.0481	0.0536	0.0553	0.0589	0.0579
STARR (75%)	KD normal	0.0767	0.0623	0.0509	0.0571	0.0647	0.0399	0.0413	0.0403	0.0424	0.0681
	KD stable	0.0169	0.0203	0.0192	0.0294	0.0304	0.0179	0.0304	0.0196	0.0212	0.0239
	AD normal	0.4667	0.4137	0.6256	0.6256	0.4667	0.4137	0.4137	0.3078	0.2480	0.4667
	AD stable	0.0746	0.0826	0.0608	0.0651	0.0631	0.0384	0.0658	0.0480	0.0502	0.0520
STARR (50%)	KD normal	0.0711	0.0639	0.049	0.0548	0.0561	0.0390	0.0422	0.0368	0.0496	0.0646
	KD stable	0.0201	0.0255	0.0237	0.0334	0.0229	0.0179	0.0198	0.0234	0.0305	0.0116
	AD normal	0.5726	0.4137	0.4667	0.3607	0.2783	0.3607	0.3607	0.2480	0.2548	0.4667
	AD stable	0.0757	0.0743	0.0495	0.0672	0.0706	0.0440	0.0463	0.0500	0.0664	0.0617

Table 5. Cont.

Reward-risk Ratio	Goodness-of-Fit statistic	Momentum deciles									
		P1 (Losers)	P2	P3	P4	P5	P6	P7	P8	P9	P10 (Winn-ers)
R-ratio (99%,99%)	KD normal	0.0536	0.0513	0.0485	0.0404	0.0565	0.0508	0.0478	0.0450	0.0572	0.0454
	KD stable	0.0189	0.0266	0.0307	0.0245	0.0270	0.0247	0.0244	0.0254	0.0271	0.0223
	AD normal	0.3078	0.4137	0.3607	0.3078	0.3078	0.2480	0.3607	0.3078	0.2566	0.3229
	AD stable	0.0512	0.0609	0.0638	0.0539	0.0588	0.0560	0.0520	0.0601	0.0616	0.0633
R-ratio (95%,95%)	KD normal	0.0543	0.0523	0.0426	0.0491	0.0413	0.0535	0.0543	0.0565	0.0476	0.0503
	KD stable	0.0225	0.0200	0.0163	0.0194	0.0182	0.0202	0.0276	0.0260	0.0262	0.0219
	AD normal	0.3078	0.3607	0.2382	0.4667	0.3607	0.3078	0.2273	0.2548	0.3191	0.4137
	AD stable	0.0514	0.0577	0.0575	0.0684	0.0409	0.0522	0.0580	0.0579	0.0630	0.0528
R-ratio (91%,91%)	KD normal	0.0600	0.0520	0.0508	0.0500	0.0526	0.0528	0.0511	0.0526	0.0551	0.0496
	KD stable	0.0237	0.0168	0.0230	0.0194	0.0227	0.0223	0.0284	0.0324	0.0286	0.0224
	AD normal	0.4137	0.3607	0.3191	0.4137	0.3078	0.3607	0.3078	0.5196	0.3607	0.3607
	AD stable	0.0733	0.0553	0.0469	0.0500	0.0520	0.0616	0.0793	0.0658	0.0587	0.0504
R-ratio (50%,99%)	KD normal	0.0566	0.0392	0.0561	0.0424	0.0519	0.0614	0.0470	0.0520	0.0457	0.0506
	KD stable	0.0219	0.0202	0.0210	0.0173	0.0264	0.0263	0.0268	0.0322	0.0188	0.0256
	AD normal	0.3078	0.2548	0.4667	0.3229	0.5196	0.2579	0.2579	0.2548	0.4137	0.4137
	AD stable	0.0532	0.0661	0.0686	0.0614	0.0534	0.0599	0.0548	0.0668	0.0608	0.0584
R-ratio (50%,95%)	KD normal	0.0627	0.0522	0.0553	0.0462	0.0504	0.0511	0.0473	0.0442	0.0560	0.0459
	KD stable	0.0266	0.0261	0.0163	0.0237	0.0383	0.0207	0.0190	0.0218	0.0293	0.0205
	AD normal	0.3191	0.6785	0.4137	0.4137	0.3607	0.3078	0.3078	0.2548	0.5196	0.6785
	AD stable	0.0582	0.0677	0.0625	0.0584	0.0770	0.0483	0.0615	0.0538	0.0591	0.0706

For each 6 month strategy, the sample of 517 stocks in the S&P universe is ranked over the period January 1996 to December 2003 into 10 equally weighted portfolios and the returns are evaluated over holding period of 6 months. The table reports the Kolmogorov distance (KD) and the Anderson-Darling (AD) statistic assuming the unconditional homoscedastic distributional model of i.i.d. decile returns. Loser (P1) is the equally weighted portfolio of 10% of the stocks with the lowest value of criteria over the past 6-months, and winner (P10) is the equally weighted portfolio of 10% of the stocks with the highest value of criteria over the past 6-months.

Table 6. Performance Evaluation of Momentum Spreads from Strategies using Cumulative Return Criterion and Alternative Criteria on Cumulative Profit, Sharpe measure and Independent Performance Measure

Stock ranking criterion	Cumulative profit	Sharpe measure	$E(X_t)/$ $CVaR_{99\%}(X_t)$
Cumulative Return	**6.9934**	0.03566	0.01398
Sharpe Ratio	2.7917	0.02098	0.00803
STARR (99%)	2.6550	0.01959	0.00758
STARR (95%)	3.6614	0.02523	0.00978
STARR (90%)	3.0497	0.02243	0.00858
STARR (75%)	3.2852	0.02476	0.00948
STARR (50%)	3.2596	0.02413	0.00927
R-ratio (99%,99%)	2.7919	**0.04980**	**0.02177**
R-ratio(95%,95%)	2.4508	0.03660	0.01550
R-ratio(91%,91%)	3.3875	0.04516	0.01872
R-ratio(50%,99%)	0.9404	0.01557	0.00660
R-ratio(50%,95%)	0.5964	0.00890	0.00366

This table reports the evaluation of momentum strategies using cumulative return benchmark criterion and risk-adjusted criteria on cumulative profit, Sharpe measure, and independent performance measure. The values in bold denote the best criterion performance for specific evaluation measure. Independent performance measure is a risk-adjusted performance measure in the form of $STARR_{99\%}$ ratio. The sample includes a total of 517 stocks in the S&P 500 universe during the period of January 1996 through December 2003.

Figure 1

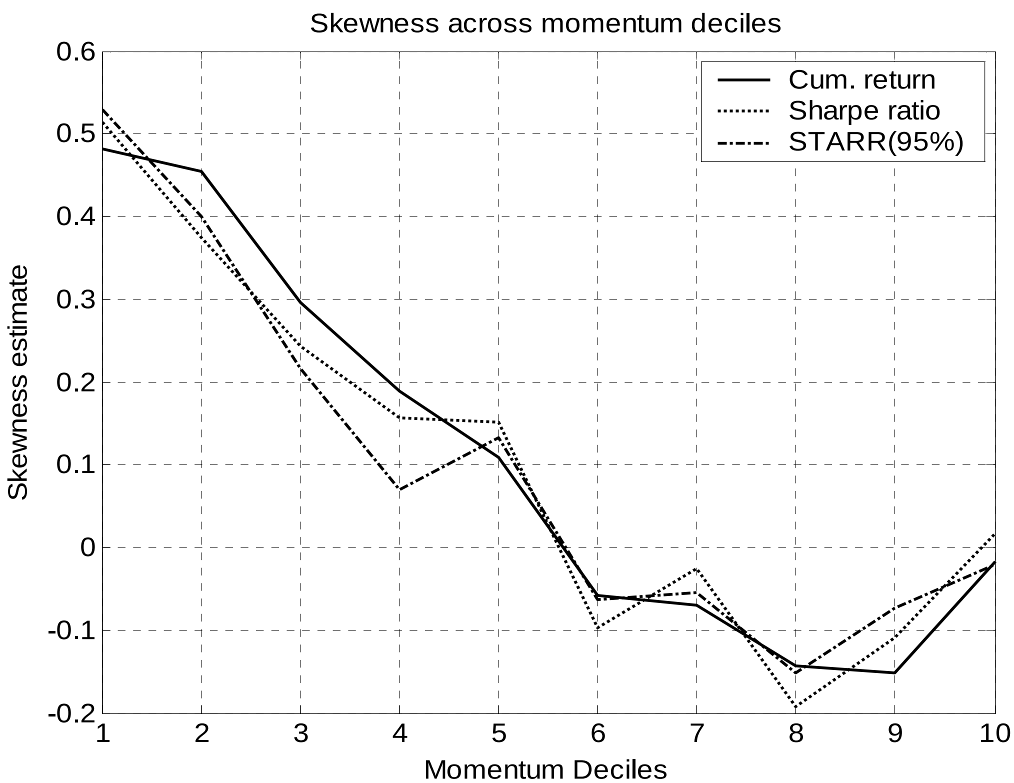
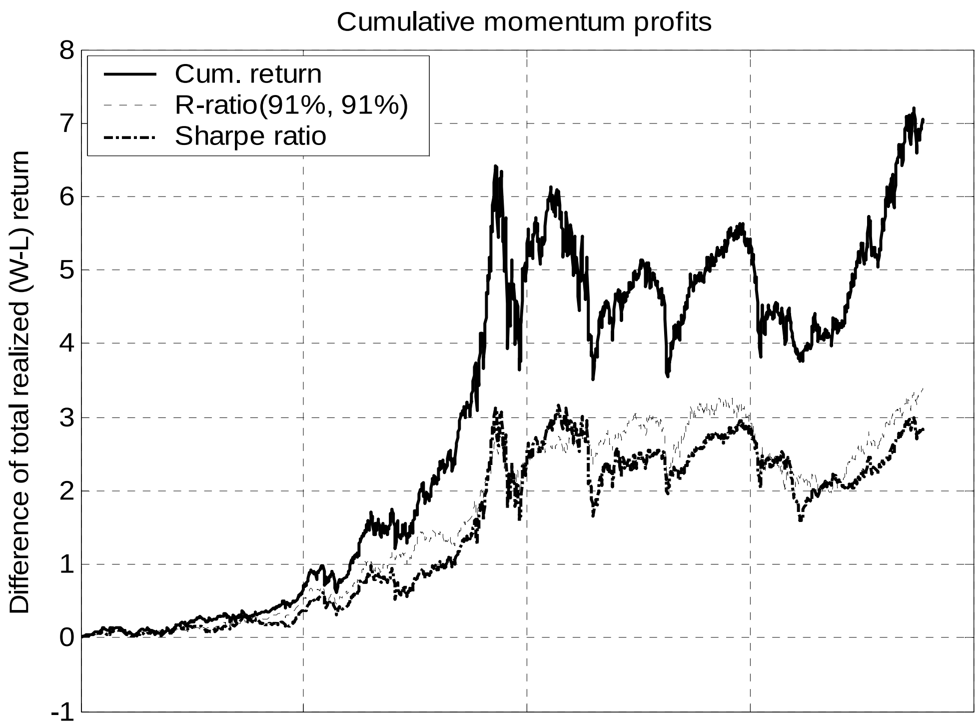

Skewness across momentum deciles
Cum. return
Sharpe ratio
STARR(95%)
Skewness estimate
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
1
2
3
4
5
6
7
8
9
10
Momentum Deciles

Figure 2



Cumulative momentum profits
Cum. return
R-ratio(91%, 91%)
Sharpe ratio
Difference of total realized (W-L) return
8
7
6
5
4
3
2
1
0
-1

Figure 3

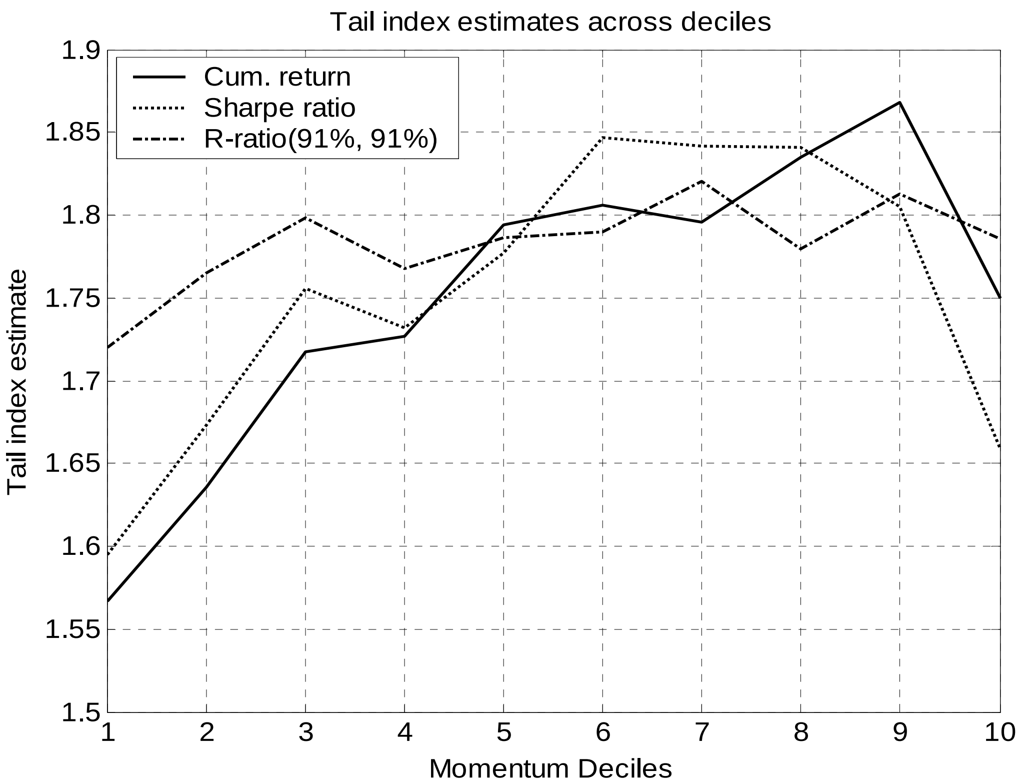
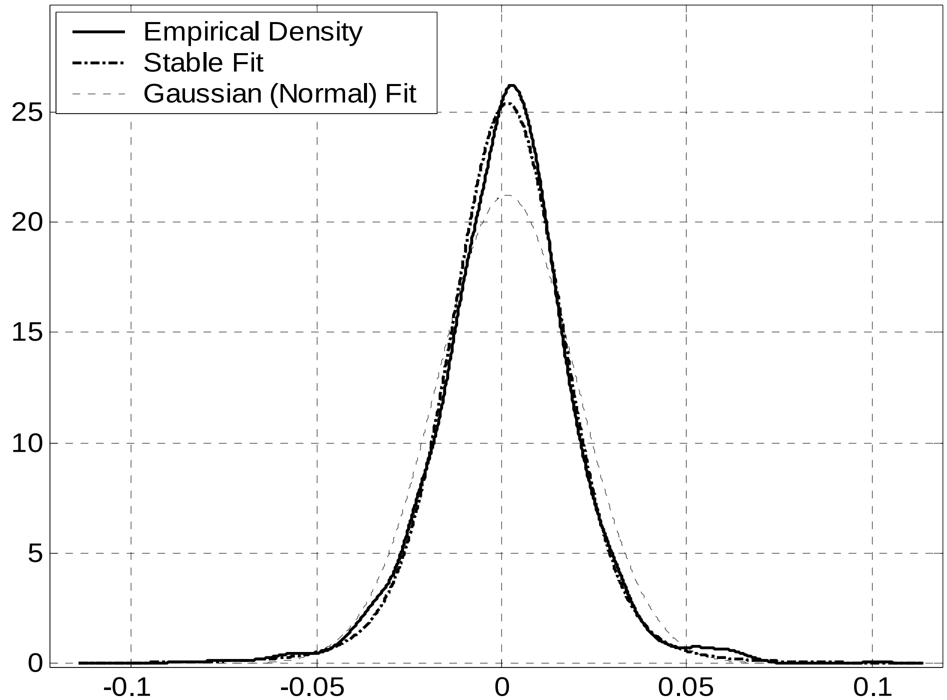
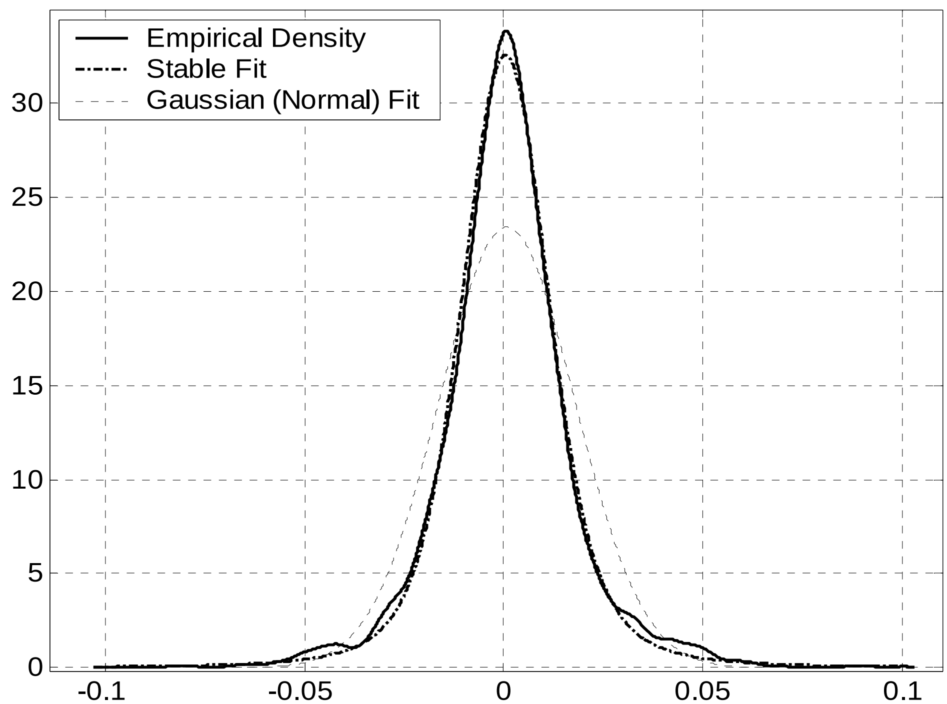

Tail index estimates across deciles
Cum. return
Sharpe ratio
R-ratio(91%, 91%)
1.9
1.85
1.8
1.75
1.7
1.65
1.6
1.55
1.5
Tail index estimate
1
2
3
4
5
6
7
8
9
10
Momentum Deciles

Figure 4



Empirical Density
Stable Fit
Gaussian (Normal) Fit
25
20
15
10
5
0
-0.1
-0.05
0
0.05
0.1

Figure 5



Empirical Density
Stable Fit
Gaussian (Normal) Fit
30
25
20
15
10
5
0
-0.1
-0.05
0
0.05
0.1

Figure 6

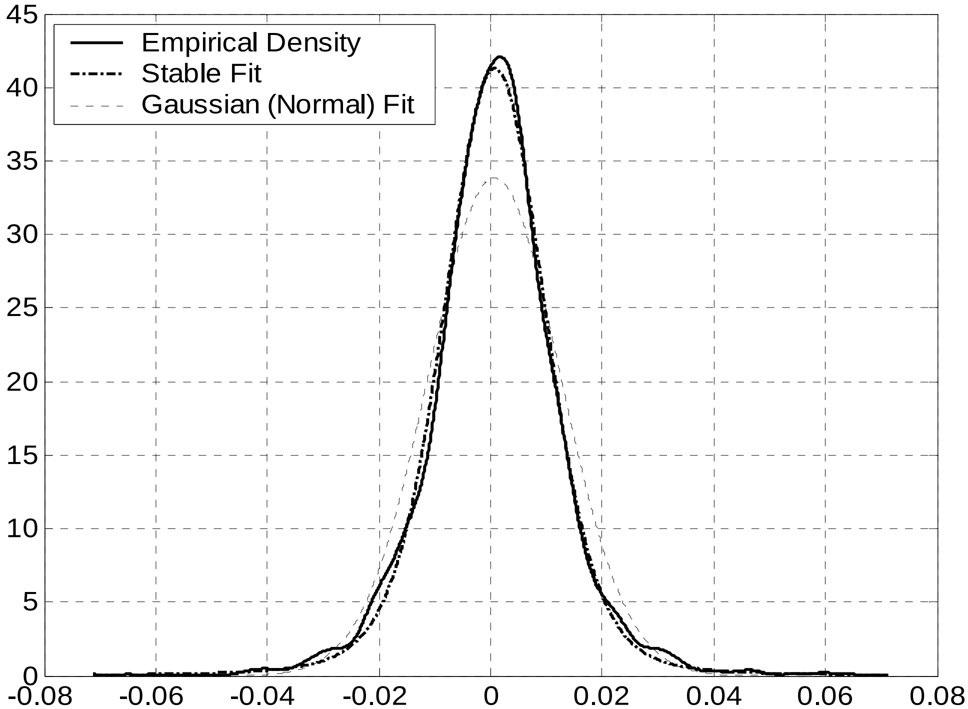
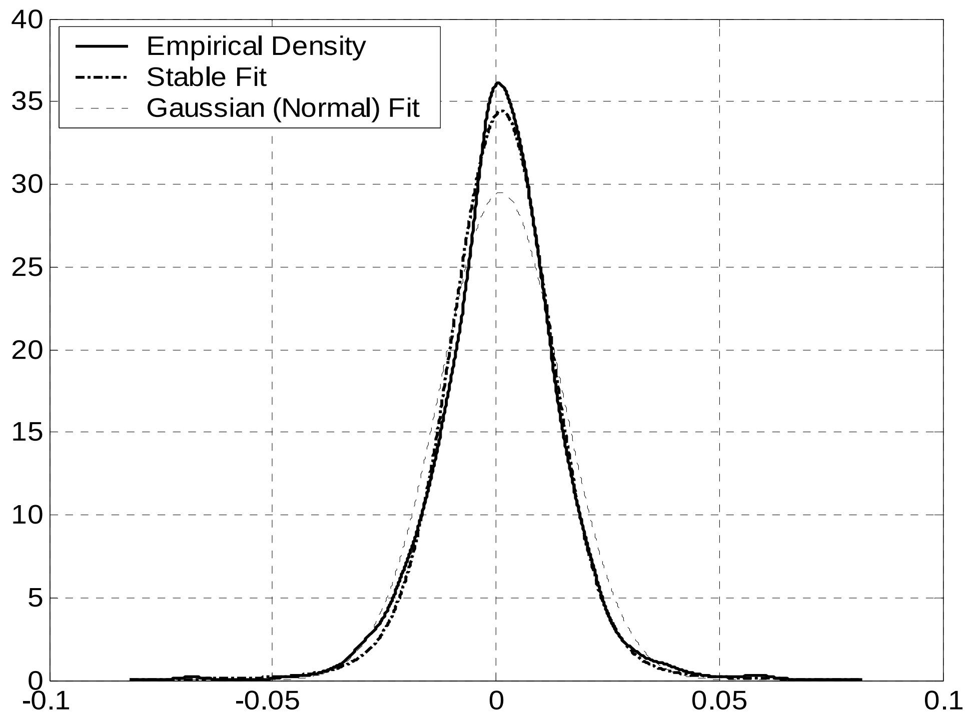

Empirical Density
Stable Fit
Gaussian (Normal) Fit
45
40
35
30
25
20
15
10
5
0
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.08

Figure 7


Empirical Density
Stable Fit
Gaussian (Normal) Fit
40
35
30
25
20
15
10
5
0
-0.1
-0.05
0
0.05
0.1