Profitability of momentum strategies: Application of novel risk/return Ratio stock selection criteria

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Abstract

In this paper, we introduce performance risk/return Ratios as new criteria for the construction of the winner and loser portfolios in momentum strategies. Our results show that momentum strategies based on novel risk/return Ratio criteria applied to previous 6-month or 12-month periods, generate positive returns over 6-month and 12-month holding periods and are more profitable than strategies based on usual cumulative or total return benchmarks. We compare strategies driven by different performance Ratios and cumulative return benchmark using independent performance measures based on a coherent risk measure (expected shortfall risk) and cumulative return over the holding period. Specifically, the R-Ratio emerges in our study as the best candidate for momentum portfolio construction. Finally, we model momentum profits in a GARCH-stable setting and validate our assumptions on the winner and loser return patterns.

Key Words: momentum strategies, risk/return Ratio criterion, GARCH-stable modeling

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1. Introduction

The momentum effect has attracted considerable attention because the application of momentum strategy is simple and its consistent profitability poses a strong challenge to the theory of asset pricing. A number of researchers have concluded that single-factor and multi-factor models such as three factor model of Fama and French (1996), fail to explain the abnormal momentum returns. A momentum effect captures the short-term (6 to 12 months) return continuation effect that stocks with high returns over the past three to 12 months tend to outperform in the future (see Jegadeesh and Titman, 1993). The simplicity of momentum strategy is based on the mechanistic decision criterion of compounded total monthly return or cumulative monthly return in selecting winners and losers over some defined ranking period.

Empirical findings on momentum strategies show that stock return continuation on horizons between 6 and 12 months is evident for the United States, Europe, and emerging markets (Jegadeesh and Titman 1993, Rouwenhorst 1998, Rouwenhorst 1999, Griffin et al. 2003) with historically earned profits of about 1% per month over the following 12 months. Although some have argued that these results provide strong evidence of "market inefficiency", others have argued that the returns from these strategies are either compensation for risk, or alternatively, the product of data mining. Recent evidence on momentum strategies and their associated profitability shows that stock return continuation on horizons between 6 and 12 months has continued in the 1990s (Jegadeesh and Titman, 2001), providing strong evidence that the results are not spurious or the result of "data snooping". However, the interpretations of the empirical findings in studies that investigate the additional possible causes and sources of momentum effect are divergent and generate further debate.

While the finding of persistence of stock return continuation effect across various markets and in different time periods has neutralized the data-mining argument that has been suggested in the literature, the macroeconomic-based explanations are still unsettled. Although Chordia and Shivakumar (2002) claim that a multifactor macroeconomic model of returns explains the momentum profits found in tests using U.S. data, the most recent evidence in studies by Griffin et al. (2003) and Cooper et al. (2004) present evidence that macroeconomic models cannot explain U.S. and international momentum profits.

In addition, Griffin et al. (2003) examine conditions among returns to momentum strategies in various countries and show that momentum profits are large and have only a weak comovement among countries, whether within regions or across continents. This fact indicates that if momentum is driven by risk, the risk is largely country specific. Moreover, Cooper et al. (2004) find that the macroeconomic model cannot forecast the time series of momentum profits out-of-sample, while the lagged returns of the market can. Therefore, the lagged return of the market is the type of conditioning information that can be relevant for predicting the profitability of momentum strategies.

In previous and contemporary studies of momentum strategies, possible effects of non-normality of individual stock returns and their risk characteristics have received little attention. To the best of our knowledge, all studies on momentum strategies utilize monthly data as a basis for ranking and evaluation of the investment period profits and

simple cumulative return as the criterion for ranking stocks into winner and loser sub-portfolios.

Given abundant empirical evidence that stock returns exhibit non-normality, leptokurtic, and heteroscedastic properties, such effects are clearly important and in interaction with criteria for portfolio construction may have an impact on momentum strategies and its profits. The implication that returns of financial assets exhibit a heavy-tailed distribution may have a significant impact on risk management and investment application such as momentum strategies. Extreme returns may occur with a much larger probability where the return distribution is heavy tailed than where it is normal. In addition, quantile-based measures of risk, such as value at risk (VaR), may also be significantly different if calculated for heavy-tailed distributions. This may have a significant impact on the evaluation of risk/return profiles of individual assets, their subsequent aggregation, and the impact on the investment decision in a risk/return framework.

It would be therefore of interest to integrate information from the non-normal data distributional properties with their impact on risk/return profile of stock returns and utilize this information to form new stock selection criteria. The aim of this paper is to outline an approach that extends existing momentum methodology by defining the selection criteria for portfolio construction within the risk/return framework. This approach is based on risk/return Ratios which include not only a return but also a risk component of individual stock returns, thereby capturing the risk/return profile of assets. In addition, to provide consistency with the most recent theoretical advances on risk measures, we focus on coherent measures of risk[1] when applying and evaluating the risk/return Ratio performance. Our approach of using novel risk/return Ratios is more convenient for daily data. It is more general than usual total or cumulative return selection criterion since it incorporates risk information into the estimation of the expected (excess) return of a particular stock. Moreover, we define a risk coherent independent performance measure for direct comparison of performance Ratios within risk/return framework.

To test the impact of Ratio-based selection criteria, we examine the returns of the winner and loser stocks in the 6 and 12 months following the formation period of 6 and 12 months. We analyze non-overlapping returns in these periods, or in other words, we rank stocks (determine winner and loser portfolios) every six and 12 months using performance Ratios as criterion. In our experiments, we examine a wide range of Ratios as criteria for momentum portfolio construction and compare their performance in terms of momentum profits and independent performance measure. In addition, we explicitly model time-series of momentum profits in a GARCH (1,1)-stable setting framework.

Our empirical findings for the small sample of German stocks investigated in this study indicate that momentum strategies with risk/return Ratios based on coherent measures of risk using daily data are indeed profitable, and these profits may exceed the profits from using common cumulative return benchmark. Specifically, the Rachev Ratio (R-Ratio) and STARR Ratio that we describe later in this paper are the clear winners when compared to traditional Sharpe Ratio and cumulative return benchmark. Our study extends the previous studies in that we include novel performance Ratio measures as the portfolio formation criterion which suits better the non-normality properties of stock

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[1] The definition of a coherent risk measure is provided later in this paper.

returns and are based on daily data. Our analysis suggests that aligning the key decision criterion of momentum strategy with the risk/return framework provides clear benefits in terms of magnitude and significance of profits in the holding period compared to cumulative return benchmark. Moreover, by modeling the daily momentum profits in GARCH-stable setting, we enable formulation of an appropriate forecasting model that can drive momentum profits by applying a specific risk/return Ratio. Insights from this modeling can also be used to provide a link to the analysis of the decomposition of momentum returns provided in other studies, a topic of future research.

The remainder of the paper is organized as follows. Section 2 provides a brief description of our data and methodology. Section 3 provides an analysis of the holding period returns and profitability of the momentum strategies we investigated. An analysis of modeling the momentum profits in GARCH-stable setting is presented in Section 4. Section 5 concludes the paper.

2. Formation of "Winner" and "Loser" Portfolios using Ratio Criteria

2.1 Data and Methodology

Our sample comprises 9 stocks traded on the German Stock Exchange and included in the DAX index (Adidas-Salomon AG, Basf AG, Bayerishe Motoren Werke AG, Continental AG, Bayer AG, Hoechst AG, Fresenius Medical Care AG, MAN AG, and Henkel KGAA). We analyze the daily returns of these stocks for the period between 27.01.1999 and 30.06.2003. For the riskless asset, we use the London interbank offered rate (Libor) in the same observation period. Although our sample of stocks is small, the results can be applied and interpreted for a larger sample of stocks.

Any momentum strategy involves decision on the (1) length of the ranking or formation period, (2) length of the holding or investment period, and (3) the ranking criterion. The ranking criterion determines winners and losers at the end of the ranking period, and the zero-investment strategy of simultaneously selling losers and buying winners produces momentum profits in the holding period. Such zero-investment strategy is applicable in practice given the regulations on proceeds from short-sales for investors (Bris *et al*., 2004). We perform ranking of stocks with returns of at least 12 months by applying the risk/return Ratio criterion to their prior 6-month of 12-month daily returns. Ranking assigns the stocks to one of the three subportfolios (1 equals lowest past performance or "Loser", 3 equals the highest past performance or "Winner"). In our study, these portfolios are equally weighted at formation and held for 6 or 12 subsequent months of the holding period; during the holding period, these portfolios are not rebalanced.

Since we perform ranking using daily data, we initially focus on the analysis of non-overlapping holding period returns in this study, implying no rebalancing within the holding period. Previous studies on momentum strategies usually report the monthly average return of *K* strategies, each starting one month apart which is equivalent to a composite portfolio in which each month 1/K of the holding is revised. We will subsequently also apply our ranking method to overlapping holding period returns with 1, 3, 6, and 9 months rebalancing. We follow the most widely used practice of reporting the results that focus on a 6-month ranking period over which raw total returns determine

winner or loser status. We use a 6-month and 12-month holding period with equal weights, and the investment rule is followed on each 6-month or 12-month ranking period. For overlapping holding period returns, the investment rule is followed (every month) such that equally weighted momentum strategies of six varying vintages are simultaneously in effect at all times. We examine the top (winner) and bottom (loser) 33% of stock return (3 shares) due to small sample size. Therefore, for each month t, the portfolio held during the investment period, months t to $t+5$ is determined by performance over the ranking period, months $t-6$ to $t-1$. Daily stock returns were calculated as

$$r(t) = \ln \frac{S(t)}{S(t-1)}$$

Formation Period   **Holding Period**

Month -5 to Month 0   Month 1 to Month 6

Month −11 to Month 0   Month 1 to Month 12

Figure 1. Time line showing formation and holding periods as combinations of momentum strategies

At the end of each subsequent formation period, we rank the stocks in our sample based on calculating Ratios using stocks' past six-month returns (month -5 to month 0). Subsequently, we group the stocks into two equally weighted portfolios based on these ranks. We consider the top 3 shares with the highest Ratio form the winner portfolios and the bottom three shares with the lowest Ratio form the loser portfolios. Each portfolio is held over the holding period of 6 or 12 months following the ranking month. The various formation and holding periods we consider are presented in the Figure 1. We performed experiments of rebalancing portfolios for the periods of 1, 3, 6, 9, and 12 months over the holding period to observe the impact on momentum profits in the holding period.

The main steps of our algorithm are the following:

Step 1. Form a matrix of excess returns (N assets, T observations)

$$\begin{pmatrix} r_1^1 - r_1^{riskfree} ,..., r_1^9 - r_1^{riskfree} \\ r_2^1 - r_2^{riskfree} ,..., r_2^9 - r_2^{riskfree} \\ ... \\ r_{1120}^1 - r_{1120}^{riskfree} ,..., r_{1120}^9 - r_{1120}^{riskfree} \end{pmatrix}$$

Step 2. Divide the data into subperiods equal to the length of the formation period, where *N* is the number of daily observations. We form the zero-investment portfolios of winners and losers at the end each formation period (e.g., for 6 months after each 161 observation) by calculating the Ratio for each share based on observations in these periods (1-160,…, 960-1,120) and ranking. Shares with the highest Ratio will constitute winner portfolio, and shares with the lowest Ratio will form the loser portfolio. Hence, given our sample of 9 stocks, we buy the top 3 shares and sell the bottom 3 shares based on the data for each ranking period.

2.2 Risk/Return Ratios

The usual approach to selecting winners and losers employed in previous studies investigating momentum strategies has been to evaluate the individual stock's past monthly returns over the ranking period (e.g., six-month monthly return for the six-month ranking period). The realized cumulative return as a selection criterion is a simple measure which does not reflect the risk-reward framework. Moreover, empirical evidence shows that individual stock returns exhibit non-normality, so that it would be more reliable to use a measure that could account for these return properties.

One of the most commonly applied measures for risk-reward framework is the Sharpe Ratio (Sharpe, 1966). This Ratio is the mean return of the trading strategy divided by its standard deviation and can be interpreted as a return/risk Ratio. However, this Ratio is unstable for low values of the denominator and does not consider the clustering of profits and losses. Moreover, as demonstrated by Leland (1999), the Sharpe Ratio is not totally reliable when applied to assets where the return distribution is non-normal.

Usual measures of risk are standard deviation and value at risk (VaR). The VaR of a random variable *X* (random profit and loss of an investment by a fixed time horizon) at level α is the absolute value of the worst loss not to be exceeded with a probability of at least α. The following is a formal definition of VaR.

Quantile, VaR: Let $\alpha \in (0,1]$ be fixed and *X* be a real random variable on a probability space $(\Omega, \Im, P)$. Define $\inf \phi = \infty$. We than call

$$q_\alpha(X) = \inf\{x \in \Re : P[X \le x] \ge \alpha\} \qquad (1)$$

the α-quantile of *X*. The VaR at confidence level *α* of *X* is VaR$_\alpha$ (*X*) = q$_\alpha$ (-*X*).

In practice, values of α close to 1 are of interest. VaR(*X*) can be interpreted as the minimal amount of capital to be put back by an investor in order to preserve his solvency with a probability of at least α. However, standard deviation lacks the ability to describe the rare events and VaR is criticized because of its inability to aggregate the risk in a legal manner. In addition, VaR is not in general subadditive and is law invariant in a very strong sense. Arztner *et al.* (1999) propose a set of properties any reasonable risk measure should satisfy. A risk measure which satisfies these properties is called *coherent*.
Risk measure can be formally described with the following definition:

Risk measure: Let $(\Omega, \Im, \mathrm{P})$ be a probability space and **V** be a non-empty set of $\Im$ –measurable real-valued random variables. Then any mapping $\rho : V \rightarrow \Re \cup \{\infty\}$ is called a risk measure.

A risk measure *ρ* is called coherent if it is monotonous, positively homogeneous, translation invariant, and subadditive. Standard deviation and VaR are not coherent measures of risk. On the other hand, the *expected shortfall*, also called *tail conditional expectation* or *expected tail loss* (ETL) is a coherent risk measure (Rockafellar and Uryasev, 2002). Another name for expected shortfall is conditional value at risk (denoted CVar). ETL (CVar) is a more conservative measure than VaR and looks at the average of all losses that exceed VaR. Formally, the expected shortfall for risk *X* and high confidence level *α* is defined as follows (Bradley and Taqqu, 2003):

Let *X* be the random variable whose distribution function F_x describes the negative profit and loss distribution (P&L) of the risky financial position at the specified horizon time *τ* (thus losses are positive). Then the expected shortfall for *X* is

$$S_\alpha(X) = E(X \mid X > VaR_\alpha(X)) \tag{2}$$

For each risk measure, there exists a performance measure to identify superior, ordinary, and inferior performance. Specifically, the Ratio between the "expected excess return" and the relative risk measure is a performance measure ρ(.) that investors wish to maximize. This performance measure is associated with a certain "market portfolio" which is based on a diverse risk perception and sometimes on a different reward perception. While it is difficult to find a "perfect performance measure" given a complexity of admissible choices, it is reasonable to assume that some performance measures take into account the common investor's opinions better than others. Biglova *et al.* (2003) compare various risk-reward performance Ratios including newly designed STARR and R-Ratio based on the criterion of maximizing the final wealth over a certain time period. The results of this study implicitly support the hypothesis that the new Ratios capture the distributional behaviour of the data (typically the component of risk due to heavy tails) better than the classical mean-variance model embodied by the Sharpe Ratio.

In order to include the risk-return framework and account for non-normality of asset returns, we also apply the STARR Ratio and R-Ratio as the criterion in forming our momentum portfolios. We analyze and compare the most popular Ratios such as Sharpe Ratio, STARR Ratio (95%), STARR Ratio (99%) and the new R-Ratio for various parameter values. A summary of the three performance Ratios is provided below::

1. *Sharpe Ratio* The Sharpe Ratio (see Sharpe, 1994) is the Ratio between the expected excess return and its standard deviation:

$$\rho(r) = \frac{E(r - r_f)}{STD_{(r-r_f)}} \tag{3}$$

where r_f is the risk free asset and STD is the standard deviation of returns of the stock r. For this Ratio it is assumed that the second moment of the excess return exists.

2. $CVaR_{(1-\alpha)\%}$ *Ratio* (*STARR Ratio*). The $CVaR_{(1-\alpha)\%}$ Ratio (see Martin, Rachev, and Siboulet, 2003) is the Ratio between the expected excess return and its conditional value at risk:

$$\rho(r) = \frac{E(r - r_f)}{CVaR_{(1-\alpha)\%}(r - r_f)} \quad (4)$$

where $CVaR_{(1-\alpha)\%}(r)$ is the $(1-\alpha)\%$ CVaR of r. $VaR_{99\%}$ is the opposite of 1% quantile that is $P(r \le -VaR_{99\%}) = 0.01$.

3.   *Rachev Ratio* (*R-Ratio*). The R-Ratio is the Ratio between the ETL of the opposite of the excess return at a given confidence level and the ETL of the excess return at another confidence level.

$$\rho(r) = \frac{ETL_{\gamma 1\%}(r_f - r))}{ETL_{\gamma 2\%}(r - r_f))} \quad (5)$$

where γ_1 and γ_2 are in [0,1]. Here, if r is a return on a portfolio or asset, then L = $-r$ presents the relative loss: $ETL_{\alpha\%}(r) = E(L / L > VaR_{\alpha\%})$, is the expected tail loss and $VaR_{\alpha\%}$ is defined by $P(L > VaR_{\alpha\%}) = \alpha$, and α is in (0,1) typically $\alpha = 0.01$.

We analyze the R-Ratio for different parameters γ_1 and γ_2. For example, R-Ratio ($\gamma_1 = \gamma_2 = 0.01$), R-Ratio ($\gamma_1 = \gamma_2 = 0.05$), and R-Ratio ($\gamma_1 = 0.5$, $\gamma_2 = 0.01$).

After calculating different performance Ratios for all stocks over a defined ranking period, we explore the performance of momentum strategies in the holding periods. We evaluate and compare performance Ratios using the aggregate wealth value at the end of the holding period and independent performance measure based on the coherent risk measure of the expected shortfall. Due to the nature of the risk/return Ratios and computational requirements, we will utilize daily data for their calculation. Following the analysis of momentum profits, we select and suggest optimal Ratio(s), which allow(s) investors to obtain momentum profits. The optimal risk/return Ratio will serve as a new criterion for constructing the momentum portfolios. [2]

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[2] It would be of interest to observe the performance of these ratios on different ranking and holding periods or their combinations. The analysis on non-overlapping holding period returns to overlapping holding period returns is the subject of future research.

3. Returns of Momentum Portfolios Formed on Risk/Return Ratio Criteria

In Panel A of Table 1 are shown the average monthly returns of the winner and loser portfolios as well as of the zero-cost, winner-loser spread portfolio for all combinations of 6- and 12-month ranking/holding horizons and for all risk/return Ratios. Monthly returns are aggregated from daily returns, given the assumption of 250 trading days in a year. The highest average winner-loser return spread (1.50% per month) for a 6-month ranking period arises for the 6-month/6-month strategy (6-month ranking period, 6-month holding period) for the R-Ratio (0.3, 0.4) and the lowest average winner-loser return spread for a 6-month ranking period arises for R-Ratio (0.01,0.01) (0.52% per month).

For the ranking period of 6-months, the average spread for the 6-month holding period compared to the 12-month holding period is higher in all cases except for the Sharpe Ratio, R-Ratio(0.05,0.05), and cumulative return. For the 6-month ranking period, the returns range from 0.52% to 1.50% per month earned by the portfolio based on (1) R-Ratio (0.01,0.01) and held for 6 months and (2) R-Ratio (0.3, 0.4) and held for 6-months. The largest return on 6-month/6-month strategy is obtained using R-Ratio (0.3, 0.4) followed by the STARR (99%), STARR (95%), R-Ratio (0.05,0.05), Sharpe Ratio, and R-Ratio (0.01,0.01).

For 6-month/12-month strategy, the largest winner-loser spread returns (1.21 % per month) are again obtained for the R-Ratio (0.3, 0.4), but the cumulative return is the second best performer (0.82% per month). The lowest average winner-loser return spread (0.18% per month) for 6-month/12-month strategy is obtained for STARR(95%) Ratio. It is interesting to note that for the 6-month/6-month strategies, the cumulative return criterion performs better than the Sharpe Ratio, R-Ratio (0.01,0.01), and R-Ratio (0.05,0.05) . For 6-month/12-month strategy, the STARR Ratio (95%) is the only Ratio that produces negative winner-loser spread return.

Panel B of Table 1 reports the average monthly returns of the winner and loser portfolios as well as the winner-loser spread portfolio for strategies using a 12-month ranking period. The highest average winner-loser return spread (1.36% per month) for the 6-month holding period arises for STARR (95%) Ratio and the lowest average (-0.13%) arises for 12-month holding period and cumulative return benchmark. The highest average winner-loser return spread (1.37% per month) for the 12-month holding period arises for the STARR (99%) Ratio. Given our data set, it seems that STARR Ratios perform better on a longer, 12-month ranking period. Compared to the results on the 6-month ranking strategy, R-Ratio (0.3, 0.4) underperforms STARR(95%) Ratio and R-Ratio (0.05,0.05).

Overall, the largest winner-loser spread is obtained on the 6-month/6-month strategy using R-Ratio (0.3, 0.4) and the lowest for 12-month/12-month strategy using cumulative return. For different combinations of ranking and holding periods, the winner-loser spread results using the same Ratios may differ and even change sign. STARR and R-Ratio (0.3, 0.4) obtain better results than the cumulative return and Sharpe Ratio for all combinations of momentum strategies.

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Table 1.Momentum portfolio returns

Panel A: Ranking Period 6 Months

Risk/Return Ratio	Holding Period		
	Portfolio	6 months	12 months
Cumulative return (Benchmark)	Loser	-0.0069	-0.01089
	Winner	0.0000645	-0.00268
	Winner-Loser	0.0070	0.00821
Sharpe Ratio	Loser	-0.00785	-0.011108
	Winner	-0.00113	-0.00346
	Winner-Loser	0.00671	-0.00891
R-Ratio (0.01,0.01)	Loser	-0.00822	-0.00998
	Winner	-0.00303	-0.00547
	Winner-Loser	0.00518	0.00450
R-Ratio (0.05,0.05)	Loser	-0.0114	-0.01180
	Winner	-0.00409	-0.00439
	Winner-Loser	0.00731	0.00741
R-Ratio (0.3,0.4)	Loser	-0.01196	-0.01110
	Winner	0.00305	0.001025
	Winner-Loser	0.01501	0.01213
STARR (95%)	Loser	-0.00635	-0.00217
	Winner	0.00339	-0.00396
	Winner-Loser	0.00974	-0.00178
STARR (99%)	Loser	-0.01303	-0.00737
	Winner	-0.00263	-0.00355
	Winner-Loser	0.01039	0.00382

Panel B: Ranking Period 12 Months

Risk/Return Ratio	Holding Period		
	Portfolio	6 months	12 months
Cumulative return (Benchmark)	Loser	-0.00933	-0.00253
	Winner	-0.00319	-0.003908
	Winner-Loser	0.00613	-0.00137
Sharpe Ratio	Loser	-0.00933	-0.00253
	Winner	-0.0021	-0.00390
	Winner-Loser	0.00723	-0.00137
R-Ratio (0.01,0.01)	Loser	-0.00537	-0.00503
	Winner	-0.00236	-0.00401
	Winner-Loser	0.00300	0.00102
R-Ratio (0.05,0.05)	Loser	-0.01046	-0.00841
	Winner	-0.00092	0.00193
	Winner-Loser	0.00953	0.01035
R-Ratio (0.3,0.4)	Loser	-0.00746	-0.00755
	Winner	0.00251	-0.00415
	Winner-Loser	0.00997	0.00339
STARR (95%)	Loser	-0.00909	-0.00788
	Winner	0.00454	0.00152
	Winner-Loser	0.01363	0.00941
STARR (99%)	Loser	-0.01167	-0.01321
	Winner	-0.00012	0.000483
	Winner-Loser	0.01155	0.01370

This table reports the daily and monthly returns for momentum portfolios based on past J-month returns, specific risk/return Ratio criterion, and held for subsequent K months. Loser (P1) is the equally weighted portfolio of 30% of the stocks with the lowest past J-month returns, and winner (P3) is for those with the highest past J-month returns. The sample includes a total of 9 stocks traded on the German Stock Exchange during the period of January 1999 and June 2003.

Figure 2 presents the graph of total realized returns of winner and loser portfolios over the entire holding period for the R-Ratio (0.3, 0.4) which gives the best performance for investors as will be shown in the following analysis. As can be seen in Figure 2, the total realized returns of the winner portfolio are always higher than the total realized returns of the loser portfolio. This means that for this particular performance Ratio, the momentum strategy works in that profits are positive over the entire observation period, and specifically in this case, the magnitude of the profits are increasing over the time horizon.
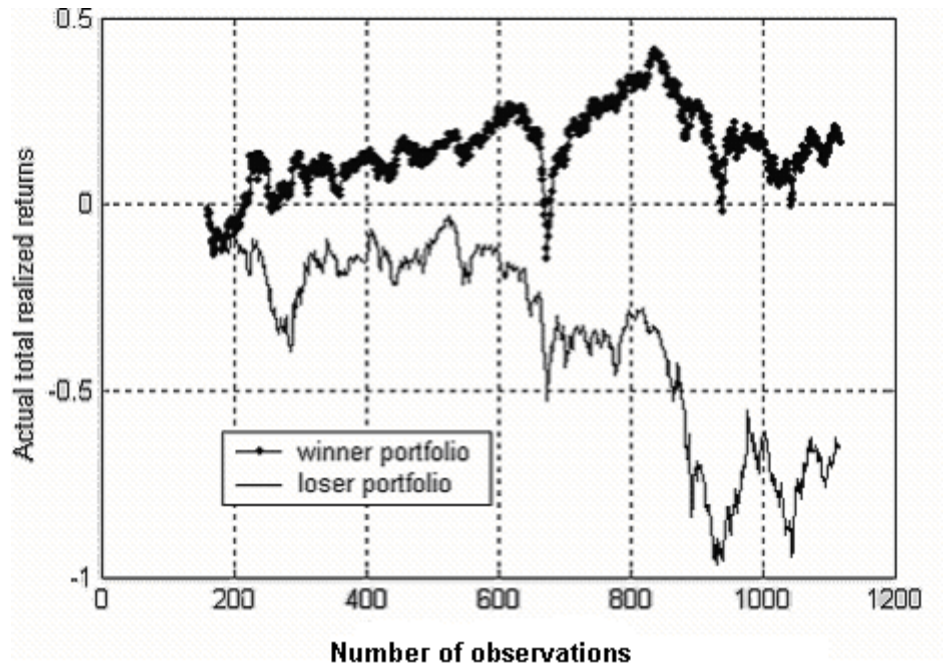

Figure 2. Actual total realized returns of winner and loser portfolios for R-Ratio(0.3, 0.4) and 6-month/6-month momentum strategy

In the remainder of this paper we will concentrate on portfolios formed on the basis of six-month daily returns that are formed at the end of the ranking period. We explore which of the performance Ratios can drive profitable momentum strategies thereby allowing us to eliminate those performance Ratios that consistently produce inaccurate results. The performance Ratios are evaluated based on a comparison of the cumulative total realized returns of the winner and loser portfolios and their difference over the entire observation period, 1999-2003. We seek the largest positive difference over this holding period.

We further evaluate the performance of the Ratios by calculating the final value of the compounded return based on the difference of the cumulative returns between the winner and loser portfolios. To illustrate, look at Figure 3 where we present the graphs of

the difference of cumulative returns of winner and loser portfolios for the four risk/return Ratios criteria and cumulative returns for the 6-month/6-month strategy over all holding periods. The R-Ratio(0.3, 0.4) clearly provides the best total return over the entire observation period. It is interesting to note that STARR(99%) Ratio matches closely the R-Ratio performance up to two 6-month holding periods before the end of the full period and then deteriorates in performance. In the last 6-month holding period, all three risk/return Ratios are similar in performance and obtain a much lower value than the R-Ratio (0.3, 0.4). The final value of compounded return for R-Ratio (0.3, 0.4) is equal to 0.8071. The cumulative return criterion provides the worst performance since it is obvious from Figure 3 that its cumulative total realized return of the difference between the winner and loser portfolios is lower than the cumulative realized return of the winner-loser spread of each other Ratio during the entire period.
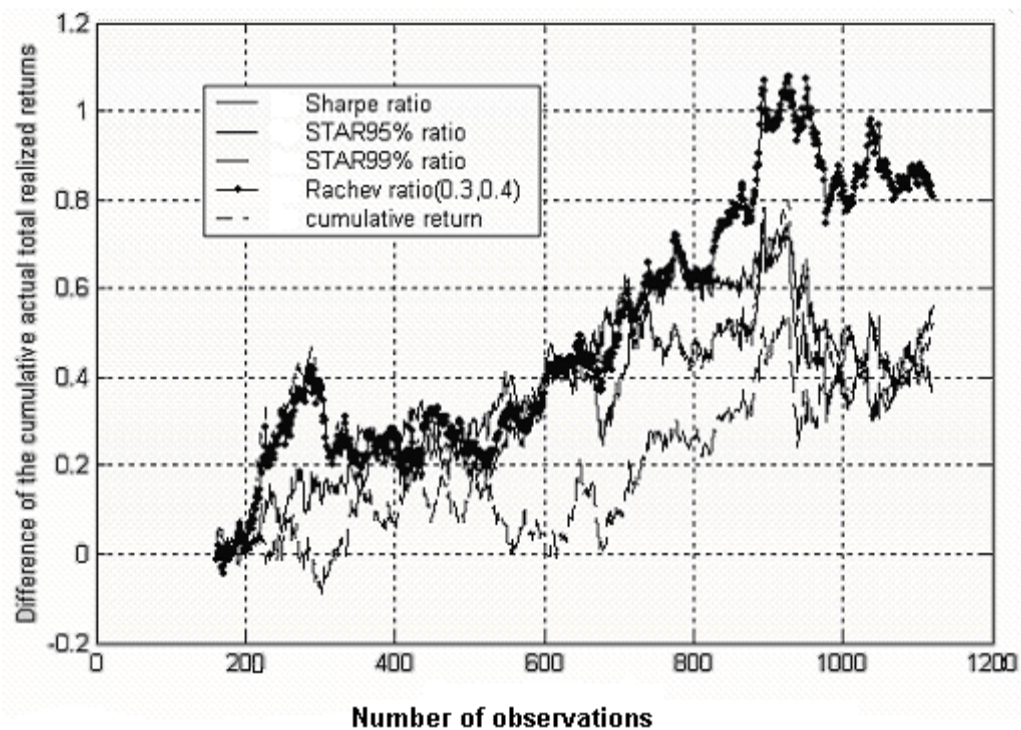

Figure 3: Cumulative total realized returns of the difference between the winner and loser portfolios for different risk/return criteria on a 6-month ranking period

We also analyze the graph of differences of returns of the winner and loser portfolios, which are actually the momentum profits. The graph of the sequence of momentum profits over the entire observation period obtained using the R-Ratio(0.3, 0.4) is presented in Figure 4.
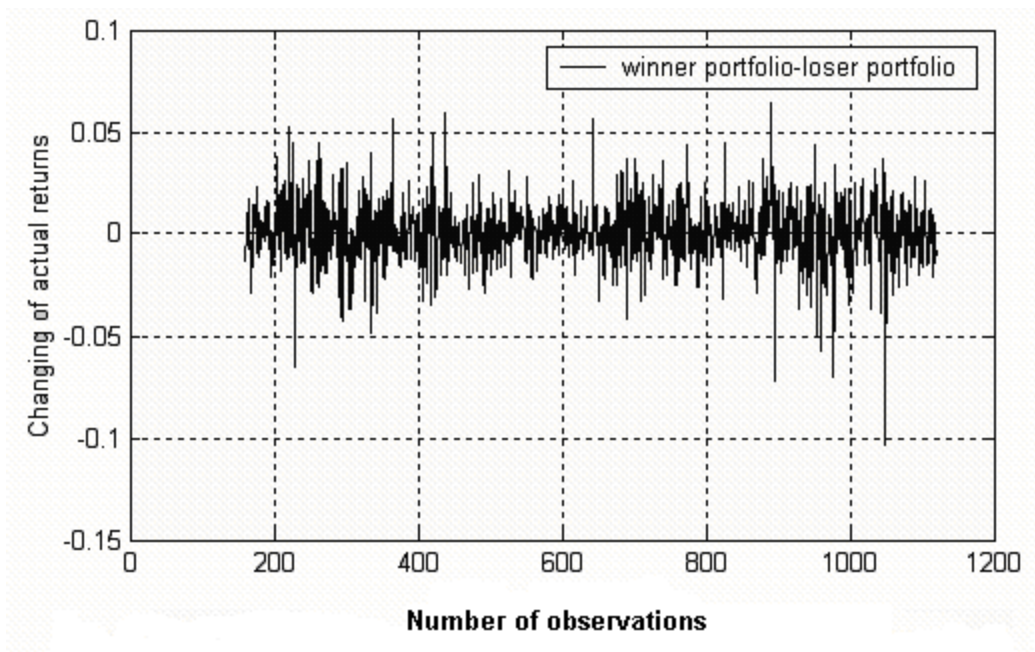
Figure 4. Momentum profits for the R-Ratio(0.3, 0.4) on a 6-month ranking period

4. GARCH-Stable Modeling of Momentum Profits

It is a well known fact supported by abundant empirical evidence that financial asset returns often posses distributions that are heavy tailed and peaked (leptokurtic). Mandelbrot (1963) and Fama and French (1963; 1965) were the first to formally acknowledge this fact and reject the standard hypothesis of normally distributed returns in favor of more general stable distribution. Since this initial work, the stable distribution has been applied to modeling both the unconditional and conditional return distributions, as well as theoretical framework of portfolio theory and market equilibrium models (see. Rachev and Mittnik, 2000).

The stable distribution $S_{\alpha,\beta}$ (σ, μ) is defined as the limiting distribution of the sum of independent and identical distribution (i.i.d.) variables. The stable distributions are described by four parameters: α - tail index, β - skewness, μ - location, and σ - scale. The α-stable, or, in short, $S_{\alpha,\beta}$, distribution, has in general, no closed-form expression for its probability density function, but can, instead, be expressed by its characteristic function. A stable distribution for a random risk factor X is formally defined by its characteristic function:

$$F(t) = E\left(e^{itX}\right) = \int e^{itx} f_{\mu,\sigma}(x)dx \,,$$

where

$$f_{\mu,\sigma}(x) = \frac{1}{\sigma} f\left(\frac{x-\mu}{\sigma}\right)$$

is any probability density function in a location-scale family for X:

$$\log F(t) = \begin{cases} -\sigma^{\alpha} |t|^{\alpha}\left(1 - i\beta \operatorname{sgn}(t)\tan\left(\frac{\pi\alpha}{2}\right)\right) + i\mu t, & \alpha \neq 1 \\ -\sigma |t|\left(1 - i\beta\frac{2}{\pi}\operatorname{sgn}(t)\log|t|\right) + i\mu t, & \alpha = 1 \end{cases}$$

A stable distribution is therefore determined by the four key parameters:

1. α (index of stability) determines density's kurtosis with $0 < \alpha \leq 2$ (e.g. tail weight)
2. β determines density's skewness with $-1 \leq \beta \leq 1$
3. σ is a scale parameter (in the Gaussian case, $\alpha = 2$ and $2\sigma^2$ is the variance)
4. μ is a location parameter (μ is the mean if $1 < \alpha \leq 2$)

Stable distributions for risk factors allow for skewed distributions when $\beta \neq 0$ and fat tails relative to the Gaussian distribution when $\alpha < 2$. For positive (negative) β, the

distribution is skewed to the right (left), and is symmetric for β = 0. We estimate the parameters of a stable distribution and approximate the stable density functions by applying a maximum likelihood estimation using Fast Fourier Transform (FFT).[3] The application of computationally demanding numerical approximation method in estimation of stable distribution is necessary, while closed-form expressions for its probability density function in general do not exist.

Based on our risk/return Ratio criteria, we analyze the differences between returns of the winner and loser portfolio. The aim is to postulate the model which can be used for *ex-ante* forecasting purposes in the holding period. We would like to evaluate how different Ratios influence the model form behavior and its predictability power. To do so, we model the momentum profits for R-Ratio(0.3, 0.4). We obtain the following parameters of the α-stable distribution: α = 1.8642, β = 0.2788, σ = 0.0105, and μ = 7.3380e-004.

For values of $1<\alpha<2$, we can calculate the expected shortfall $CVAR_{99\%}$ and $\dfrac{E(X_t)}{CVAR_{99\%}(X_t)}$ of the sequence and compare these values for different performance Ratios. The best performance Ratio is the one that attains the highest value of $\dfrac{E(X_t)}{CVAR_{99\%}(X_t)}$ which serves as the independent performance measure. For the specific case of R-Ratio (0.3, 0.4), we obtain CVAR = 0.0616, and $\dfrac{E(X_t)}{CVAR_{99\%}(X_t)} = 0.0120$.

We assume that the time series of momentum profits (differences of winner and loser returns) exhibits an autocorrelation structure in the second-order moments; that is, it is heteroscedastic and exhibits periods of varying volatility. We therefore model momentum profits by a generalized autoregressive conditional heteroscedastic GARCH(1,1) model. Accordingly, a GARCH(1,1) model is estimated jointly with a conditional mean model for the return process and fitted to the series. The following GARCH specification is used:

$$y_t = c + \varepsilon_t \tag{7}$$

$$\sigma_t^2 = \omega + \beta_1 \sigma_{t-1}^2 + \alpha_1 \varepsilon_{t-1}^2, \tag{8}$$

where y_t is the momentum profit at time t, σ_t its conditional variance measured at time t, c is constant (mean), and $\{\varepsilon_t\}$ is a white noise process. We assume here that momentum payoff series do not have a zero conditional mean.

The return process is captured by the specification in the conditional mean given by equation (7), while the GARCH(1,1) process given by equation (8) tracks the conditional volatility in returns. The GARCH model extends the mean equation (7) by assuming that $\varepsilon_t = z_t \sigma_t$, where z_t is standardized residual, $z_t \sim N(0,1)$. The residuals conditional on past information are assumed to be normally distributed. The

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[3] For details of the estimation procedure, see Rachev and Mitnik (2000).

ARCH/GARCH models of conditional volatility allow for both volatility clustering (periods of large volatility) and for heavy tails. It turns out that a GARCH-type model driven by normally distributed returns implies unconditional distributions which themselves possesses heavy tails. Thus, GARCH models and α-stable distribution might be viewed as competing hypothesis.

The parameters of GARCH(1,1) model are estimated for each ranking period and based on these parameters, we simulate returns in the holding period and then estimate the parameters of simulated returns. The estimated parameters are presented in Table 2.

Table 2: GARCH(1,1) estimation results for momentum profits obtained using R-Ratio(0.3, 0.4) on 6-month ranking period.

Parameter	Value	Std. Error	t-stat
c	0.00068526	0.00047499	1.4427
ω	1.3932e-005	4.9334e-006	2.8240*
α_1	0.0917	0.0174	5.2439*
β_1	0.8595	0.0305	28.1752**

The table reports estimated coefficients, standards errors and t-statistics. * indicates statistically significant at the 5% level. ** indicates statistically significant at the 1% level.

So, our estimated GARCH (1,1) model is:

$$y_t = 0.00068526 + \varepsilon_t \qquad\qquad (9)$$

$$\sigma_t^2 = 0.3932e - 005 + 0.85953\sigma_{t-1}^2 + 0.09170\varepsilon_{t-1}^2 \qquad (10)$$

To estimate the expected shortfall of the conditional distribution of the difference returns, we apply a three-step procedure. In the first step, we use a GARCH(1,1) model for the (conditional) volatility of the difference return series and estimate the model parameters $\omega, \alpha_1, \beta_1$. Based on the parameters of the estimated GARCH(1,1) model, the second step involves generating the new sequence of model innovations $(y_{t-n+1}/\sigma_{t-n+1},...,y_{t-1}/\sigma_{t-1}, y_t/\sigma_t)$. This gives a sequence of simulated returns, which we test for normality. In the third step, the stable distribution is used to model the tails of the distribution of those innovations. The stable fit is then applied to the innovations and the parameters of the stable distribution are obtained.

The last graph shown in Figure 5 represents the sequence of simulated returns. We test the normality assumption in the GARCH(1,1) model given in (9) and (10). For that purpose, we analyze the sequence $\dfrac{y_t observed}{\sigma_t estimated}$.
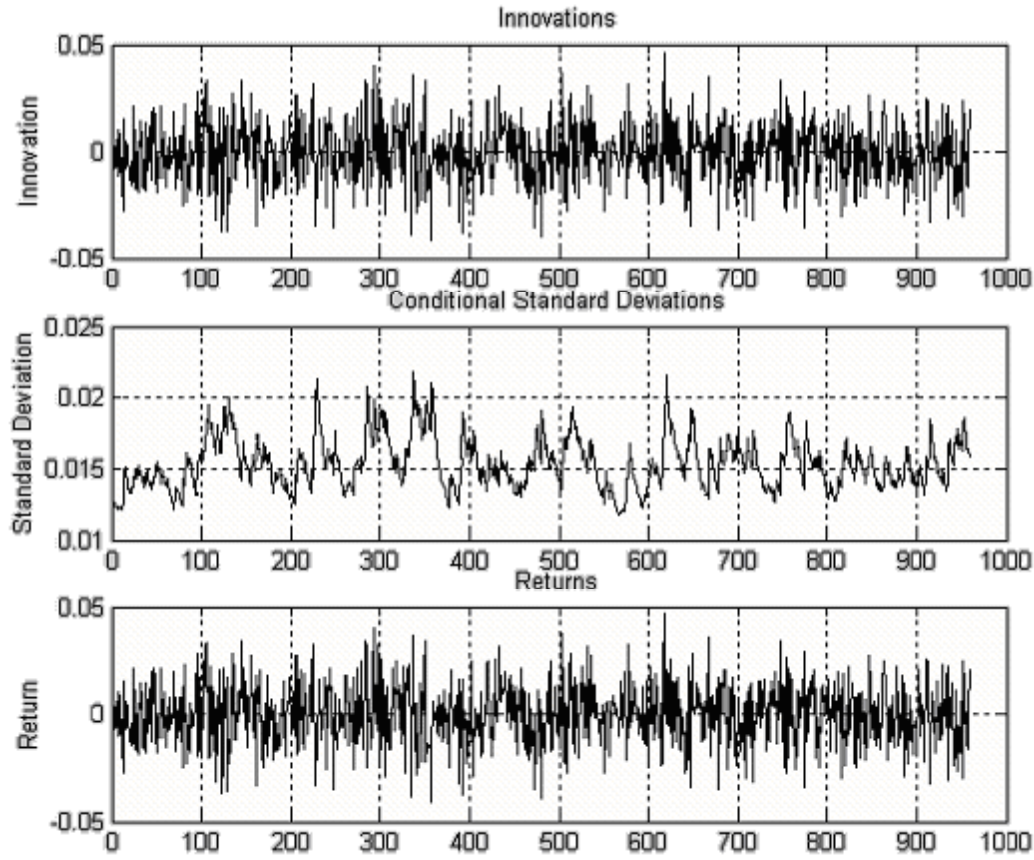
Figure 5. Innovations from a GARCH model over the 6-month ranking and holding periods

Traditionally, the innovation distribution is assumed normal. Figure 6 represents the graph of distribution density of this sequence and shows that this assumption may still underestimate the tails of the loss portion of the distribution. Notice that the lower (loss) tail of the innovations is still heavier than the normal distribution.

We observe that the innovations exhibit heavier tails than that the normal. The fit of stable non-Gaussian distribution is now applied to the innovations and the following parameters are obtained: α = 1.8774, β = -0.1927, σ = 0.6466, and μ = 0.0480.

Since $\alpha > 1$, the usual GARCH (1,1) model is still applicable (Rachev *et a*l., 2003). Therefore, our postulated model is valid so that we can generate innovations for GARCH and by using (9) and (10) we can generate simulations of y_t returns in the holding period. For practical purposes, we can repeat the modeling procedure for the ranking period and specific performance Ratio, and use it for forecasting of momentum profits over the holding periods.
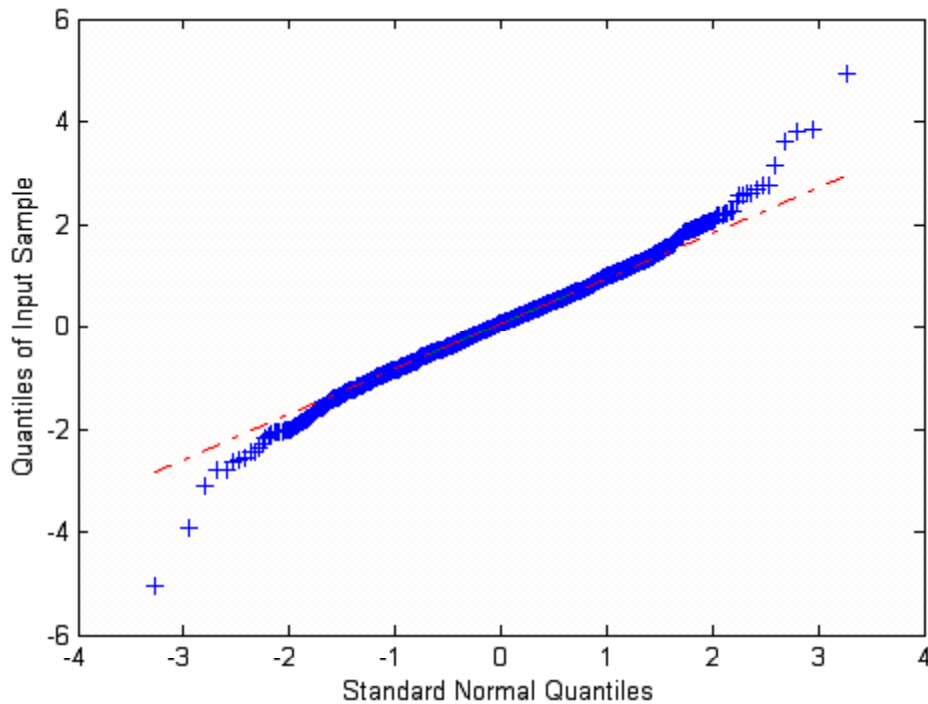
Figure 6. Quantile-quantile (QQ) plot of the conditionally normal GARCH(1,1) *ex post* model innovations with the normal returns. Normal returns are on the abscissa and innovations on the ordinate. Returns are expressed as %. Innovations are from Figure 5.

The values of α-stable parameter are calculated for all analyzed Ratios and are in the range between 0.18010 and 0.19272. The highest values of α, 1.9272 and 1.9130, are obtained for STARR(95%) and STARR(99%) Ratio, respectively. Table 3 shows the index of stability for all Ratios along with the final value of the compounded return and independent performance measure. The best performance in terms of independent performance measure and compounded return is achieved for R-Ratio (0.3, 0.4), followed by R-Ratio (0.25, 04) and R-Ratio (0.2, 0.4). The Sharpe Ratio performs poorly on the data we analyzed.

Our results clearly indicate that by applying new risk/return Ratio measures as the criterion for momentum portfolio construction, we are able to achieve better overall return and achieve optimal risk-return performance as measured by the independent performance measure. It seems that the R-Ratio and STARR Ratio are able to better capture the non-normality features of the data and transfer this into an optimal reward measured by the independent performance measure. These results also confirm the deficiency of the Sharpe Ratio as the effective risk/reward performance measure. In addition, by modeling the momentum profits as a GARCH-stable model, we are able to use the model for *ex-ante* forecasting of momentum strategy.[4]

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[4] The precise formulation of this model with respect to the length of the ranking and holding periods, specifics of applied risk/return ratio criterion, and the impact of the overlapping holding-period returns is the topic of our future research.

Table 3. Performance of risk/return Ratios on and independent performance measure for the 6-month/6-month momentum strategy over the entire observation period.

Risk/Return Ratio	Final value of compound return	α of stable distributions of momentum profits	$\dfrac{E(X_t)}{CVAR_{99\%}(X_t)}$
Sharpe Ratio	0.3686	1.8637	0.0056
R-Ratio (0.01,0.01)	0.2789	1.8641	0.0055
R-Ratio(0.05,0.05)	0.3928	1.8618	0.0062
R-Ratio(0.5,0.01)	0.2039	1.8568	0.0035
R-Ratio (0.1,0.1)	0.4865	1.8853	0.0080
R-Ratio (0.2,0.2)	0.6757	1.8794	0.0119
R-Ratio (0.3,0.3)	0.6801	1.8696	0.0114
R-Ratio (0.4,0.4)	0.7105	1.8640	0.012
R-Ratio (0.5,0.4)	0.4477	1.8494	0.0070
R-Ratio (0.4,0.5)	0.5373	1.8728	0.0087
R-Ratio (0.4,0.3)	0.4892	1.8716	0.0080
R-Ratio (0.2,0.4)	0.6783	1.8763	0.0116
R-Ratio (0.25,0.4)	0.7177	1.8632	0.0123
R-Ratio (0.3, 0.4)	0.8071	1.8519	0.0136
R-Ratio (0.35,0.4)	0.6980	1.8642	0.0117
R-Ratio (0.5, 0.5)	0.5585	1.8594	0.0088
R-Ratio (0.6,0.6)	0.3106	1.8303	0.0046
R-Ratio (0.7, 0.7)	0.3558	1.8228	0.0053
R-Ratio (0.8,0.8)	0.2167	1.8010	0.0032
R-Ratio (0.9,0.9)	0.3997	1.8198	0.0063
STARR (95%)	0.5239	1.9272	0.0122
STARR (99%)	0.5591	1.9130	0.0114

5. Conclusions

In this study, we introduce risk/return Ratios using daily data as the criterion in constructing momentum portfolios. Proposed risk/return Ratios take into account risk component of the individual stock returns and their empirical non-normality characteristics. We evaluate and compare a wide range of risk/return Ratios and cumulative return benchmark using an independent performance measure based on the risk coherent measure of expected shortfall. Our results confirm that the R-Ratio and STARR Ratio capture well the features of the data and obtain the best performance in terms of cumulative return and value of independent performance measure while the Sharpe Ratio underperforms on the same measures. In addition, when applying the performance Ratios as the portfolio construction criterion, we demonstrate that the momentum profits obtained can be modeled in a GARCH-stable setting and such model can possibly be used for forecasting momentum profits in the holding period. This evidence is preliminary and requires further research.

The implications of our results are twofold. First, we show that empirical facts and characteristics of the data require more complex risk/reward measures as has been to date in studies of momentum strategies. By applying performance Ratios on daily data and aligning the selection of stocks with their risk/return profile as the driver of a momentum strategy, we are able to obtain the same, if not larger and more persistent, momentum profits over the holding periods as compared to previous strategies based on cumulative return. Second, by utilizing a daily data in a GARCH-stable setting, we set the stage for postulating models which will help to gain further insights and provide a link to theoretical explanations of the momentum effect reported in the literature.

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