Risk and Return in Momentum Strategies: Profitability from Portfolios based on Risk-Adjusted Stock Ranking Criteria

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ABSTRACT

Risk-adjusted stock ranking criteria applicable when stock returns are not normally distributed are able to generate more profitable momentum strategies than those based on usual cumulative or total return criterion. These alternative risk-return ratio criteria conform to properties of coherent risk measures and, in different form, capture the risk of the tail distribution. Replacing the cumulative return by the risk-adjusted criterion, we also utilize the ratio as the objective function in the portfolio optimization problem and obtain optimal risky winner and loser portfolios. Our results are robust to transaction costs for both equal-weighted and optimized-weighted strategies. In particular, our alternative ratios outperform the cumulative return and the Sharpe ratio across all strategies measured by total realized return and independent performance measures over the observed period.

In an efficient securities market, an appropriate risk-return trade-off is the cornerstone for informed investors in their portfolio selection of securities. The finding that returns exhibit momentum behavior at intermediate horizons is at odds with market efficiency. Additionally, the lack of consistent risk-based explanation established momentum effect is the most challenging asset pricing anomaly. It continues to draw considerable attention from academic researchers and practitioners because the application of a momentum strategy is simple and its consistent profitability poses a strong challenge to the theory of asset pricing.

A momentum effect captures the short-term (6-12 months) return continuation effect that stocks with high returns over the past three to 12 months continue to perform well relative to other stocks in future periods (Jegadeesh and Titman, 1993). Empirical findings on momentum strategies show that stock return continuation for horizons between 6 and 12 months is evident for the United States, Europe, and emerging markets (Jegadeesh and Titman 1993, Rouwenhorst 1998, Griffin, Ji, and Martin, 2003) with historically earned profits of about 1% per month over the following 12 months. Although some have argued that these results provide strong evidence of "market inefficiency," others have argued that the returns from these strategies are either compensation for risk or the product of data mining. The momentum effect does not appear to be driven by data-mining or "data snooping" that has been suggested in the literature; the persistence of stock return continuation is found across various stock markets outside the U.S. (Griffin, Ji, and Martin, 2003) and in samples from different time periods (Jegadeesh and Titman 1993, 2001). However, the interpretations of the empirical findings in studies that investigate the additional possible causes and sources of momentum effect are divergent and this has generated further debate.

What puzzles many researchers in explaining the momentum effect is the inability to provide a consistent risk-premium based explanation for momentum profits linking two seemingly inexplicable phenomena of persistence of momentum profits at intermediate horizons and dissipation of profits at longer horizons. In addition, consistent and coherent theoretical explanation should reconcile the appearance of momentum effect in individual stocks and its absence at the market level where market returns exhibit reversal.

The macroeconomic-based risk explanations are still unsettled. A number of researchers have concluded that single-factor and multi-factor models with factor mimicking portfolio returns such as three factor model of Fama and French (1996), fail to explain the abnormal momentum returns. Although Chordia and Shivakumar (2002) claim that a multifactor macroeconomic model of returns explains the momentum profits found in tests using the U.S. data, the most recent results in studies by Griffin, Ji, and Martin (2003) and Cooper, Gutierrez, and Hameed (2004) present evidence that macroeconomic models cannot explain U.S. and international momentum profits. Lewellen and Nagel (2004) examine the momentum effect with the conditional CAPM model using short-window regressions without state variables and find that momentum portfolios' alphas remain large, statistically significant and close to their unconditional estimate. In addition, they observe that momentum portfolios have the least persistent betas, presumably reflecting their higher turnover, with betas also highly correlated with past market returns. Griffin, Ji, and Martin (2005) observe that while market correlations are much higher in down markets than in up markets, momentum correlations are low in both market conditions and momentum profits do not differ appreciably between up and down markets suggesting that momentum may be useful in international portfolio management. The challenge to understanding the sources of momentum profits, nature and components of momentum portfolio risk, and rewards that investors demand for bearing momentum portfolio risk, remains.

The simplicity of momentum strategy is based on the mechanistic decision criterion for stock performance evaluation and ranking. Previous and contemporary studies of momentum strategies focus on simple cumulative return or total return as the criterion for ranking stocks into winner and loser portfolios and use monthly data for ranking and evaluation of the holding period profits (Jegadeesh and Titman, 1993, 2001; Grundy and Martin, 2001). The aim of this paper is to extend existing momentum methodology by defining the stock selection criteria within the risk-return framework. We introduce risk-adjusted criteria in the form of alternative risk-return ratios. Our risk-return ratio approach to stock ranking is more general while it combines a risk profile of a stock based on some risk measure with the estimation of the expected (excess) return of a stock over a certain period of time. Traditional risk-adjusted measures or reward-risk measures such as the Sharpe ratio can be applied to this task. However, the Sharpe ratio is unstable for low values of the denominator and is not reliable when the underlying data deviate from normality assumption. By embedding the criterion within the risk-return framework, we are implicitly faced with the choice of an appropriate risk measure. To provide consistency with the relevant concept of risk and definition of risk measures, we consider recent advances on risk measures and devise alternative ratios using coherent measures of risk. We focus our analysis on the risk-return ratio criteria including the traditional Sharpe ratio and alternative ratios defined using a coherent risk measure of expected shortfall.

We apply risk-return ratios at the individual security level in order to drive the stock ranking process (construction of momentum portfolio) and at the portfolio level in order to evaluate and optimize the risk-return profile of the winner and loser portfolio. We investigate whether the application of risk-adjusted criteria with balanced risk-return performance can generate more profitable strategies than those based on a simple cumulative return criterion which serves as a benchmark. Moreover, by introducing risk-return ratios as portfolio selection criteria, we are able to postulate a portfolio optimization problem with a ratio as an objective function. Therefore we devise an optimized-weighted strategy that creates optimal risky winner and loser portfolios. In order to compare and evaluate the performance of different alternative ratios, we define a coherent risk independent performance measure.

Our approach to portfolio construction using risk-return criteria is based on daily data. In previous and contemporary studies of momentum strategies, possible effects of non-normality of individual stock returns and their risk characteristics have not been explored in detail. Given abundant empirical evidence that stock returns exhibit non-normality, leptokurtic, and heteroscedastic properties, such effects are clearly important and may have a considerable impact on investment strategies and their risk assessment. Extreme returns may occur with a much larger probability where the return distribution is heavy tailed than where it is normal. In addition, quantile-based measures of risk, such as value at risk (VaR), may also be significantly different if calculated for heavy-tailed distributions. Tokat, Rachev, and Schwartz (2003) show that two distributional assumptions (normal and stable Paretian [Insert Footnote 1 here]) may result in considerably different asset allocations depending on the objective function and the risk-aversion level of the decision maker. By using the risk measures that pay more attention to the tail of the distribution, preserving the heavy tails with the use of a stable model makes an important difference to the investor who can earn up to a multiple of the return on the unit of risk he bears by applying the stable model. More recently, Rachev, Ortobelli, and Schwartz (2004) examine the optimal allocation obtained assuming respectively either Gaussian or the stable non-Gaussian distributional index returns and find that the stable non-Gaussian approach is more risk preserving than the normal one. Thus, consideration of a non-normal return distribution plays an important role in the evaluation of the risk-return profile of individual stocks and portfolios of stocks.

We also assess the economic significance of momentum strategies by considering the cost of trading and its impact on profitability. Grundy and Martin (2001) analyze the average turnover of stocks in winner and loser portfolios and calculate that at round trip transaction costs of 1.03%, the profits on their momentum strategies are driven to zero. Korajczyk and Sadka (2004) estimate transaction costs based on trading models with price impacts and observe that trading costs cause a large decline in the apparent profitability of equal-weighted strategies, since their performance measures decrease dramatically even when a relatively small investment is considered. We also examine the usual equal-weighted strategy and an optimized-weighted strategy derived from a portfolio selection model in which the risk-return ratio criteria are used as the objective. Unlike Korayczyk and Sadka who solve the optimization problem for the liquidity trading model only for the winner portfolio using simplifying assumption that all assets in the winner portfolio have the same expected return, we perform direct optimization of weights in both the winner and loser portfolios using a risk-return ratio criterion. In addition, we outline an approach as to how the optimization can be extended to devise a strategy including all stocks considered for ranking.

Our empirical findings for a large sample of the U.S. stocks investigated in this study indicate that momentum strategies with alternative risk-return criteria based on expected shortfall using daily data are more profitable than strategies using common cumulative return criterion or traditional Sharpe ratio risk-return criterion. Some alternative ratios deliver robust performance across various strategies, which differ in the length of ranking and holding periods. We also find that an optimized-weighted strategy produces consistently better results than an equal-weighted strategy for every criterion, with the alternative ratios again obtaining the best performance. Our paper differs from previous and contemporary papers in several important aspects. First, we extend the empirical methodology in that we replace cumulative return and consider risk-return ratio as the momentum portfolio formation criterion. Second, we deploy a coherent measure of risk (expected shortfall) and define alternative reward-risk criteria at the individual stock and portfolio level as opposed to the traditional Sharpe ratio in the mean-variance setting. Third, we apply the criteria using daily data as opposed to monthly data commonly used in other momentum studies. Fourth, using ratio criteria as the objective function, we formulate a portfolio optimization problem that leads to strategies that we find are more profitable and more immune to the transaction cost impact than the equal-weighted strategies.

Our analysis suggests that risk-adjusted momentum strategy using alternative ratios is more profitable than "cumulative return benchmark" strategy. We also find that the alternative criteria obtain better performance than the Sharpe ratio for each specific momentum strategy studied. A likely reason is that alternative ratios capture better the non-normality properties of stock returns and provide more precise estimation of the risk and reward than the traditional mean-variance measures such as the Sharpe ratio. The results indicate that aligning stock ranking criterion as the key decision criterion of momentum strategy with the risk-return framework provides clear benefits in terms of magnitude and significance of profits in the holding period compared to the cumulative return benchmark. Even after accounting for transaction costs, equal-weighted and optimized-weighted strategies using alternative ratio criteria remain profitable.

The remainder of the paper is organized as follows. Section I provides a definition of risk-adjusted criteria and alternative risk-return ratios. Section II describes our data and methodology. Section III provides an analysis of the momentum portfolio returns for equal-weighted and optimized-weighted strategy before and after accounting for transaction costs. The performance of various momentum strategies using different risk-adjusted criteria is evaluated in Section IV. Section V concludes the paper.

I. Risk-Adjusted Criteria for Stock Ranking

The usual approach to selecting winners and losers employed in previous studies on momentum strategies has been to evaluate the individual stock's past monthly returns over the ranking period (e.g., six-month monthly return for the six-month ranking period). The realized cumulative return as a selection criterion is a simple measure, which does not include the risk component of the stock behavior in the ranking period [Insert Footnote 2 here]. While formally a zero-investment, analyses done on historical data show that the momentum strategy is not riskless. In addition, empirical evidence shows that individual stock returns exhibit non-normality, so it would be more reliable to use a measure that could account for these properties.

One of the most commonly applied risk-reward measures is the Sharpe ratio (Sharpe, 1966). This ratio is the mean return of an individual stock divided by its standard deviation and can be interpreted as a risk-return ratio. It measures the variability of returns – the higher the value of the Sharpe ratio, the higher the ratio of return to variability, the less the variability of returns. Wider interpretation of this ratio includes a benchmark so that the Sharpe ratio of any investment or portfolio return X (random variable) is defined as the expected excess return ($X - b$) over a benchmark b divided by the standard deviation of ($X - b$). A high expected Sharpe ratio is preferred to a low expected Sharpe ratio, because it implies a considerably higher expected return than the benchmark for relatively little extra risk. On the contrary, low expected Sharpe ratio offers little extra expected return relative to the greater risk entailed. However, this ratio is unstable for low values of the denominator and does not consider the clustering of profits and losses. Although the Sharpe ratio is fully compatible with normally distributed returns (or, in general, with elliptically distributed returns), it may lose reliability in interpreting risk adjusted performance as the normal distribution assumption is relaxed (Leland, 1999). Recently, Bernardo, and Ledoit (2000) show that Sharpe ratios are misleading when the shape of the distribution is far from normal. Considering the general case where the asset returns have a stable non-Gaussian distribution (Rachev and Mittnik, 2000), the application of the Sharpe rule will lead to incorrect investment decisions.

Considering the non-normal properties of stock returns in the context of momentum trading, we aim to obtain risk-adjusted performance criterion that would be applicable to the most general case of a non-Gaussian stable distribution of asset returns [Insert Footnote 3 here]. It is a well established fact based on empirical evidence that asset returns are not normally distributed, yet the vast majority of the concepts and methods in theoretical and empirical finance assume that asset returns follow a normal distribution. Since the initial work of Mandelbrot (1963) and Fama and French (1963; 1965) who rejected the standard hypothesis of normally distributed returns in favor of more general stable Paretian distribution, the stable distribution has been applied to modeling both the unconditional and conditional return distributions, as well as theoretical framework of portfolio theory and market equilibrium models (see Rachev, 2003). While the stable distributions are stable under addition (i.e., a sum of stable random variables is also a stable random variable), they are fat-tailed to that extent that their variance and all higher moments are infinite.

A. Expected Shortfall

Usual measures of risk are standard deviation and value at risk (VaR). The VaR at level $(1-a)100\%$, a ? $[0,1]$, denoted $VaR_{(1-a)100\%}(r)$ for an investment with random return r, is defined by $Pr(l > VaR_{(1-a)100\%}(r)) = a$, where $l=-r$ is the random loss, that can occur over the investment time horizon. In practice, values of a close to zero are of interest, with typical values of 0.05 and 0.01. VaR is not "sensitive" to diversification and, even for sums of independent risky positions, its behavior is not as we would expect (Fritelli and Gianin, 2002). The deficiencies of the VaR measure prompted Arztner, Delbaen, Eber, and Heath (1999) to propose a set of properties any reasonable risk measure should satisfy. They introduce the idea of coherent risk measures, with the properties of monotonicity, sub-additivity, translation invariance, and positive homogeneity.

Standard deviation and VaR are not coherent measures of risk. In general, VaR is not subadditive and is law invariant in a very strong sense. On the other hand, the *expected shortfall*, also called *conditional VaR*, or *expected tail loss* (ETL) is a coherent risk measure (Arztner, Delbaen, Eber, and Heath, 1999; Rockafellar and Uryasev, 2002, Bradley and Taqqu, 2003). ETL is a more conservative measure than VaR and looks at how severe the average (catastrophic) loss is if VaR is exceeded [Insert Footnote 4 here]. Formally, ETL is defined by

$$ETL_{a100\%}\ (r) = E(l|l > VaR_{(1-a)100\%}\ (r)), \qquad (1)$$

where r is the return over the given time horizon, and $l = -r$ is the loss. $ETL_{a100\%}\ (r)$ is also known as *Conditional VaR* [Insert Footnote 5 here], denoted by $CVaR_{(1-a)100\%}\ (r) = ETL_{a100\%}(r)$ (see Martin, Rachev, and Siboulet, 2003).

The ETL is a subadditive, coherent risk measure and portfolio selection with the expected shortfall can be reduced to a linear optimization problem (see Martin, Rachev, and Siboulet, 2003, and the references therein).

B. Alternative Risk-Return Ratios

Various risk-reward performance measures and ratios have been studied in the literature. Recently, Biglova, Ortobelli, Rachev, and Stoyanov (2004) provide an overview of such risk-reward performance measures and compare them based on the criterion of maximizing the final wealth over a certain time period. The results of their study support the hypothesis that alternative risk-return ratios based on the expected shortfall capture the distributional behavior of the data better than the traditional Sharpe ratio. In order to include the risk profile assessment and account for non-normality of asset returns, we apply the alternative Stable-Tail Adjusted Return ratio (STARR) and the Rachev (R) ratio [Insert Footnote 6 here] as the criteria in forming momentum portfolios. We analyze and compare the traditional Sharpe ratio with alternative STARR and R-ratios for various parameter values that define different level of coverage of the tail of the distribution. A summary of the three risk-return ratios is provided below:

1. *Sharpe Ratio.* The Sharpe ratio (see Sharpe, 1994) is the ratio between the expected excess return and its standard deviation:

$$r(r) = \frac{E(r - r_f)}{s_{(r-r_f)}} \qquad (2)$$

where r_f is the risk-free asset and s_r is the standard deviation of r. For this ratio it is assumed that the second moment of the excess return exists.

2. *CVaR*$_{(1-a)100\%}$ *Ratio* (*STARR*$_{(1-a)100\%}$ *Ratio*). The CVaR$_{(1-a)100\%}$ ratio (also known as *STARR*$_{(1-a)100\%}$ *Ratio*, see Martin, Rachev, and Siboulet, 2003) is the ratio between the expected excess return and its conditional value at risk:

$$r(r) = \frac{E(r - r_f)}{CVaR_{(1-a)100\%}(r - r_f)} =: \text{STARR}_{(1-a)100\%} \qquad (3)$$

where $CVaR_{(1-a)100\%}(r) = ETL_{a100\%}(r)$, see (1).

3. *Rachev Ratio* (*R-ratio*). The R-ratio with parameters a and ß is defined as:

$$r(r) = \frac{ETL_{a100\%}(r_f - r)}{ETL_{b100\%}(r - r_f)} = :\text{R-ratio}(a, ß) \qquad (4)$$

where a and ß are in [0,1]. Here, if r is a return on a portfolio or asset, and $ETL_a(r)$ is given by (1). We analyze the R-ratio for different parameters a and ß. For example, R-ratio (a = 0.01, ß = 0.01), R-Ratio (a = 0.05, ß = 0.05), and R-ratio (a = 0.5, ß = 0.5).

The idea behind the R-ratio construct is to try to simultaneously maximize the level of return and get insurance for the maximum loss. R-ratio as given by (4) can be interpreted as the ratio of the expected tail return above the level, divided by the expected tail loss. In other words, this is a ratio that awards extreme returns adjusted for extreme losses. Note that the STARR ratio is the special case of the R-ratio since $CVaR_{(1-a)100\%} = ETL_{a100\%}$. For example, STARR(95%) = R-ratio(1, 0.05). We analyze the R-ratio for different parameters a and ß, which we may optimally calibrate in the back-testing analysis.

Thus we apply risk-return ratios as alternative criteria for the creation of the winner and loser portfolios in momentum strategies. After forming the portfolio of winners and losers based on ranking calculated values of specific ratio criteria for considered stocks in the ranking period, we evaluate the performance of momentum strategy in the holding period. Specifically, we analyze winner–loser spread returns, their risk-adjusted performance and final wealth value of the momentum portfolio. Due to the definition of the risk-return ratios and computational requirements, we use daily data for their calculation. Following the analysis of momentum profits, we identify the best performing ratios, which allow investors to pursue a profitable momentum strategy.

II. Formation of Momentum Portfolios using Risk-adjusted Criteria

A. Data and Methodology

Any momentum strategy involves decisions regarding (1) the length of the ranking or formation period, (2) the length of the holding or investment period, and (3) the ranking criterion. The strategy is implemented by simultaneously selling losers and buying winners at the end of the ranking period, and held over the holding period. Regarding the length of the ranking and holding period, we define the "*J*-month/*K*-month" strategy (or simply J/K strategy) that evaluates returns over the past *J*-months and holds the position for the next *K*-months. The ranking criterion determines winners and losers based on prior returns in the ranking period, and the zero-investment, self-financing strategy generates momentum profits in the holding period. Such zero-investment strategy is applicable in international equity investment management practice given the regulations on proceeds from short-sales for investors (Bris, Goetzmann, and Zhu, 2004).

We apply our strategy first to non-overlapping *K*-month investment horizons. This means that positions are held for *K*-months after which the portfolio is re-constructed (rebalanced). We also apply our strategy to one-month holding horizons, in which case the portfolio is rebalanced monthly. With a holding period of *K*-months, the return on the portfolio strategies consists of equal-weighted average returns from the strategies implemented at the end of the previous *K*-months. Some studies include a certain period of time between ranking and holding period in order to avoid microstructure effects (i.e., bid/ask bounce, short-term reversals). We also perform analysis with this time gap of one month. Since we use a risk-return criterion approach, the impact of the waiting period between formation and investment period may be different from that considering usual cumulative return criterion.

We consider momentum strategies based on the ranking periods of 6 and 12 months and the subsequent holding periods of the same length. We rank the stocks by applying the risk-adjusted criterion to the daily returns in the period of *J*-months. Therefore, for each month *t*, the portfolio held during the investment period, months *t* to *t*+5 for *K* = 6 or *t* to *t* + 11 for *K* = 12, is determined by performance over the ranking period, months *t*-6 to *t*-1 for *J* = 6 or *t*-12 to *t*-1 for *J* = 12. Following the usual convention, we rank the stocks in ascending order and assign them to one of the ten subportfolios (deciles). "Winners" are those stocks with the top 10% ranking-period returns and "losers" are those stocks with the lowest 10% ranking period returns (with returns of at least 12 months by applying the risk-return ratio criterion to their prior 6-month of 12-month daily returns.). Winner and loser portfolios are equally weighted at formation and held for 6 or 12 subsequent months of the holding period; during the holding period, these portfolios are *not* rebalanced.

Our sample consists of a total of 382 stocks included in the S&P 500 index in the period January 1, 1992 to December 31, 2003. Due to addition and removal of stocks from the index in the observed period, we restricted the whole universe of the S&P stocks to a smaller sample of stocks with equal and complete return history. This enables uniform analysis and evaluation of applied strategies. We analyze the daily returns of stocks in the observed 12-year time period. For the riskless asset, we use daily observations of the one-month London interbank offered rate (US$ Libor) in the same observation period.

Since we perform stock ranking using daily data, we initially focus on the analysis of non-overlapping holding period returns in this study, which implies rebalancing the whole portfolio every *K*-month periods. We also consider the investment periods of one month so that the portfolio is rebalanced every month. Consecutive formation periods then have a *J*-month overlap. We denote this strategy *J*-month/1-month (J/1) strategy. Previous studies on momentum strategies (Jegadeesh and Titman 1993, 2001; Korajczyk and Sadka, 2004) usually report the monthly average return of *K* strategies, each starting one month apart which is equivalent to a composite portfolio in which each month 1/*K* of the holding position is revised.

Daily stock returns were calculated from the time series as

$$r_t = \ln \frac{P_t}{P_{t-1}}$$

in the observed period, where P_t is the (dividend adjusted) stock price at t.

We can summarize our procedure for implementation of momentum strategy as follows:

Step 1. Form a matrix of excess returns (N assets, T observations)

$$\begin{pmatrix} r_1^1 - r_1^{riskfree}, \ldots, r_1^N - r_1^{riskfree} \\ r_2^1 - r_2^{riskfree}, \ldots, r_2^N - r_2^{riskfree} \\ \ldots \\ r_T^1 - r_T^{riskfree}, \ldots, r_T^N - r_T^{riskfree} \end{pmatrix}$$

Step 2. Divide the data into sub-periods equal to the length of the formation period, where T is the number of daily observations. Form the zero-investment portfolios of winners and losers at the end of each formation period of J-months by calculating the ratio for each stock based on observations in this period and ranking the stocks. Stocks with the highest ratio values will constitute winner portfolio, and stocks with the lowest ratio will form the loser portfolio.

Step 3. Evaluate the performance of the winner and loser portfolios and of the zero-cost strategy (taking a long position in the winner top decile portfolio and a short position in the bottom loser decile portfolio) at the end of each holding period, or on a monthly basis when monthly rebalancing is applied.

B. Optimization of Winner and Loser Portfolios Based on Risk-Return Criteria

Following our ranking procedure and determination of equal-weighted winner and loser portfolios, we can further improve the performance of momentum strategy by optimizing the weights within winner and loser portfolios. To do so, at the rebalancing time points, we solve two optimization problems where the risk-return ratio ? (Sharpe ratio or R-ratio) is used as an objective function in the optimization. The optimal risky portfolios for extreme deciles are given by the portfolio that maximizes the criterion measure ?(.) for winners and minimizes the same measure for losers. Generally, this measure is a ratio between the "expected excess return" of a portfolio and a risk measure of portfolio return.

Therefore, for any risk-return criterion ?(.), we compute the optimal winner portfolio of the following optimization problem:

$$\max_x r(x'r) \quad s.t. \quad \sum_{i=1}^{n} x_i = 1;\ x_i \geq 0;\ \ i = 1,\ldots,N \tag{5}$$

To determine the portfolio weights in the loser portfolio we solve the following optimization problem:

$$\min_y r(y'r) \quad s.t. \quad \sum_{i=1}^{n} y_i = 1;\ y_i \geq 0;\ \ i = 1,\ldots,N \tag{6}$$

where ? is the ratio criterion, x_i and y_i are optimized weights in the winner and loser portfolios respectively, and n equals the number of stocks in winner or loser portfolio.

By solving two optimization problems, we adjust the proportion of stocks in the winner and loser portfolio according to the weights obtained. We calculate the profits of an optimized-weighted strategy for the Sharpe ratio and alternative ratios over the holding periods as in the equal-weighted strategies. Finally, we evaluate momentum profits adjusted for transaction costs at the rebalancing time points.

For different ranking criteria, we will obtain different optimal portfolios because all the admissible choices are not uniquely identified by only two parameters. Therefore, true optimal momentum portfolio composition of the each criterion measure ?(.) is based on a diverse risk perception and sometimes on a different reward perception. Our portfolio selection approach based on reward-risk criteria follows the approach of Markowitz (1952) which reduces the portfolio choice to a set of two criteria, reward and risk. In general, this approach is not consistent with the formal approach based on an axiomatic model of risk preferences with expected utility portfolio selection. Further research can provide a link with expected utility portfolio selection under various assumptions for distributional properties of stock returns.

Optimization strategy can be extended along several directions. We can relax the assumption on the fixed number of winners and losers [Insert Footnote 7 here], and devise a strategy that invests in the entire universe of selected S&P500 stocks. In the first optimization step, we consider an optimization objective to maximize the expected excess return for a portfolio including the whole universe of sample stocks, with constraints specifying the target level of risk desired for the portfolio return, and weights that are constrained within an interval symmetrical around zero. Stocks from this optimization model can be then sorted into winner and loser portfolios based on the value of the weights (positive or negative). Subsequently, optimal zero-cost portfolio of winners and losers can be then obtained by minimizing the ETL of the momentum portfolio return with constraint on the expected target return of spread (winner-loser), additional constraints on the target level of risk for the momentum portfolio, and the usual constraints including bounds on the minimal weights and the number of positions. Finally, the transaction cost can be directly included in the optimization.

C. Transaction Costs and Momentum Portfolio Performance

Trading costs are critical to analysis of active portfolio management and can be interpreted as the cost of implementing an investment strategy. In general, trading costs are decomposed into two major components: explicit costs and implicit costs, which can differ significantly in size and ease of measurement. Explicit costs are the direct costs of trading, such as broker commission costs and taxes. Implicit costs represent indirect trading costs, the major one being the price impact of the trade. They are difficult to measure since there are no accounting charges and reports of implicit costs. Measures such as bid/ask spread estimates are not useful since they fail to capture the fact that large trades [Insert Footnote 8 here] may move price in the direction of a trade (Domowitz, Glen, and Madhavan, 1999).

Several studies on momentum strategies have considered the impact of transaction costs on momentum returns. Jegadeesh and Titman (1993) consider a 0.5% one-way transaction cost, and find that the risk-adjusted return of the momentum trading rule is 9.29% per year, which is reliably different from zero. Assumed transaction cost of 0.5% per trade in their study is conservative and based on Berkowitz, Logue, and Noser (1978) estimate of one way transaction costs of 23 basis points for institutional investors. In analyzing the persistence in mutual fund performance, Carhart (1997) estimates that trading reduces performance by approximately 0.95% of the trade's market value. He concludes that transaction costs consume the gains from following a momentum strategy in stocks. Grundy and Martin (2001) examine a 6/6 strategy with monthly rebalancing and one-month gap between formation and investment period, and determine the average turnover probabilities of 39.94% and 36.23% for winners and losers at the end of investment month respectively [Insert Footnote 9 here]. In their more recent August 1966 to July 1995 period, they find the round-trip cost that would remove the significance of (three-factor Fama and French model) risk-adjusted return is 1.46% and the round-trip cost which would absorb profits completely is 1.94%. In other words, only an investor whose round-trip costs are less than 1.5% would conclude that his net profits are statistically significant. For the raw spread return, the corresponding round-trip costs are 0.20% and 1.03%, respectively.

Lesmond, Schill, and Zhou. (2003) analyze a 6/6 strategy over the period January 1980 to December 1998 using the CRSP monthly data and find that cross-sectionally, the stocks that generate large momentum returns are precisely the stocks with high trading costs. They use four measures of transaction costs: spread estimates, mean direct effective spread and mean Roll effective spread, commission estimates, and total trading cost estimate based on limited dependent variable (LDV) estimate. The LDV measure [Insert Footnote 10 here] provides a more comprehensive estimate of the cost of trading by implicitly including not only the spread component but also the implied commissions, immediacy costs, short-sale costs, and at least some of the price impact costs.

The Lesmond, Schill, and Zhou (2003) trading cost estimates represent the mean round-trip cost for trading the stocks within the respective portfolios and are used to examine the momentum profitability after trading cost. The authors strongly reject the equality of trading costs across momentum portfolios. Specifically, the trading cost estimates for P1 (losers) are from 30% to 75% larger than those of P2 (medium portfolio). Lesmond, Schill, and Zhou also distinguish estimates based on 100% turnover and actual turnover in P1 (loser) and P3 (winner) portfolios. For the re-examined Jegadeesh and Titman (1993) strategy that they analyze, they report the mean proportions of stocks retained in the loser and winner portfolios in the subsequent holding period to be 22.7% and 15%, respectively. For their analysis of Jegadeesh and Titman (2001) strategy, they find that the average six-month momentum profits after adjustment for LDV estimates and direct effective spread plus commission are 1.115 % (t-statistics of 0.74) for 100% turnover and 2.199 % (t-statistics of 1.59) for actual turnover. Lesmond, Schill, and Zhou report trading cost estimates adjusted down for the fraction of positions retained and conclude that the extraordinary high trading cost observed for momentum strategies result from both the high trading frequency of strategy execution, as well as the costly nature of the specific securities traded.

Korajczyk and Sadka (2004) examine the profitability of long positions in winner-based momentum strategies after accounting for the cost of trading including the price impact. They analyze several models of trading costs, including measures of proportional and non-proportional (price impact) costs and estimate them using intraday data [Insert Footnote 11 here]. Similar to Lesmond, Schill, and Zhou (2003), Korajczyk and Sadka show that losers are much less liquid than winners and that they comprise stocks that have extreme past underperformance, and are biased to small firms, which may be difficult to short sell. In addition, Korajczyk and Sadka investigate the performance of a liquidity-weighted portfolio rule that maximizes, under simplifying assumptions, post-price impact expected return on the portfolio. Korajczyk and Sadka argue that (liquidity-conscious) portfolios, which attribute more weight to more liquid stocks, would potentially earn high net average returns. After incorporating transaction costs, the results indicate that proportional spread costs do not eliminate statistical significance of momentum profits. Considering the performance after price impacts, the results for the

11/1 strategy with one-month gap between formation and investment period (applied to NYSE traded firms) show that profits of the equal-weighted strategy disappear quickly, and abnormal returns for the value-weighted strategies are driven to zero with investment portfolios larger than $2 billion. However, for the liquidity-weighted strategy, abnormal returns are driven to zero only after approximately $5 billion is invested. Korajczyk and Sadka conclude that the trading costs are crucial for equal-weighted strategies, since their performance measures decrease dramatically even when a relatively small investment is considered.

For our analysis,, we use an estimate of the (one-way) total trading costs that averages 0.78% of the value of the traded stock. These estimates are based on the analysis of mutual funds' equity trading costs for a sample of 165 funds (Chalmers, Edelen, and Kadlec, 2002). Chalmers, Edelen, and Kadlec consider direct costs of spread costs and brokerage commissions in their analysis of mutual fund trading as well as tax costs due to realization of capital gains as an indirect cost. They find that fund returns (measured as raw returns, CAPM-adjusted returns, or Carhart four-factor-adjusted returns) are significantly negatively related to both expense ratios and trading costs estimates [Insert Footnote 12 here]. Annual spread cost for each fund are estimated as the product of the dollar value of each trade multiplied by the effective spread estimate (summed over all trades for the funds each quarter and divided by the value of the fund's assets/equity).

Chalmers, Edelen, and Kadlec directly quantify trading costs and find that, as a function of assets under management, spread costs average 0.47% and brokerage commissions average 0.30% annually. It means that the mean transaction cost (one-way) per unit of invested capital is 0.0078 and the same cost occurs in short selling. On average funds spend 0.78% of their assets on trading each year. Moreover, there is a considerable variation across funds in trading costs, with the 59 basis points difference between the 25th and 75th percentile [Insert Footnote 13 here].

We apply a simple model to evaluate the impact of trading costs. We denote by W_k the total value of zero-investment portfolio which is constructed at the beginning of the period k of length K [Insert Footnote 14 here]. Every K months we sell the "loser" portfolio and invest the proceeds in the winner portfolio, so that winner and loser portfolios are equal-weighted. We estimate total return of a zero-investment momentum strategy over a defined period. The estimated net adjusted return of momentum strategy for transaction costs is calculated by taking into account a one-way transaction cost *c* (for buying or selling), that is proportional to the actual value of portfolio's long or short position. For a transaction cost *c*, an estimate of one-way cost of 0.0078% and 0.0048% of the actual trade value based on Chalmers, Edelen, and Kadlec (2002) is considered. The median transaction cost (one-way) is 0.78% while the 75th percentile is 1.075% and the 25th percentile is 0.485%. Our assumption for a one-way transaction costs is plausible given the recent empirical evidence.

At the beginning of the first holding period, we form a zero-investment portfolio and incur an overall transaction cost with a value that depends on the number of necessary transactions in the winner and loser portfolios. Each change in a winner (loser) portfolio involves selling (closing out) the stocks leaving the portfolio and buying (re-shorting) the stocks entering the portfolio, at the round-trip cost. In a realistic setting, the turnover of positions will not be 100%. Some stocks will remain in the winner and loser portfolios from one holding period to another so that the entire position does not need to be closed. In addition, a number of changes (or stocks retained) in the winner portfolio may differ from number of changes (or stocks retained) in loser portfolio over the ranking period [Insert Footnote 15 here]. In other words, "replenishment" of winner and loser portfolios with stocks may occur at different rates, so that in executing the strategy the momentum investor can save a fraction of the costs on the short and long positions by holding the positions of retained stocks in the same portfolio into the next period.

We focus therefore on observing the adjustment of momentum profits at the end of the holding period for transaction costs incurred in the preceding ranking period based on the above assumptions. When investing in a winner portfolio and selling the stocks in a loser portfolio, an investor incurs a total proportional cost *c*. Then, the estimated momentum portfolio returns W_k in the holding period *k*, adjusted for transaction costs are

$$W_k = w_k - l_k - 2\left[\frac{a_k}{N_w} + \frac{b_k}{N_L}\right] * \frac{c}{100} \qquad (7)$$

where w_k and l_k are the cumulative returns of the winner and loser portfolio over the holding period k, $k = 1$, respectively. a_k and b_k is the number of changes in the winner and loser portfolio after the k-*th* ranking period (i.e., ranking period k is preceding the holding period k) respectively. $a_k = N_W$ and $b_k = N_L$, $N_W = N_L$ and N_W and N_L is the number of the stocks in the winner or loser portfolio (10% of observed stocks) respectively. c is assumed one-way transaction cost.

The final wealth of the portfolio over all holding periods can be obtained using the recursive formula for cumulative return

$$CW_k = CW_{k-1} + W_k \tag{8}$$

where CW_k is the cumulative return after k holding periods and W_k is the momentum portfolio return adjusted for transaction costs in the holding period k. We assume that $CW_0 = 0$, so that in the first ranking period we incur only upfront transaction costs for the formation of the zero-investment portfolio for the first holding period.

By tracking the actual turnover within the winner and loser portfolio for each ranking and holding period, we obtain more precise estimation of the incurred transaction costs, as compared to method of Grundy and Martin (2001) which is based on average turnover probabilities and provides only approximate estimation of the levels of transaction costs. As in other trading strategies, there is an obvious tradeoff between profitability and turnover. Since we expect the risk-adjusted criteria will have higher turnover than a cumulative return criterion, transaction costs can exert more influence on some ratio criteria than on the other.

III. Profitability of Momentum Strategy based on Risk-Return Ratio Criteria

A. Momentum Profits before Adjustment for Transaction Costs

For each strategy and each ranking criterion, we report the average monthly returns aggregated from daily data [Insert Footnote 16 here]. Unlike other studies that report t-statistics for average monthly returns, we calculate Kolmogorov-Smirnov (K-S) statistics [Insert Footnote 17 here] to account for non-normality in daily returns. We also report the results of the final wealth value of winner and loser portfolio and their difference. Table I shows the average monthly returns (in excess of the risk-free rate) of winner and loser portfolios as well as of the zero-cost, winner-loser spread portfolios for strategies using 6 or 12 month ranking and holding period for all considered ratios and cumulative return criterion. The highest average winner-loser spread (0.86% per month) for the 6/6 strategy arises for STARR(50%) ratio, and the lowest (-0.043% per month) for the STARR(99%) ratio. The average raw return for the 6/6 strategy using cumulative return is (insignificant) 0.79% per month. Among the R-ratios, the best performance provide the R-ratio (0.05, 0.05) and the R-ratio(0.3, 0.4) with the average spread of 0.77 % and 0.73% per month, respectively. The Sharpe ratio is among the worst performers with the spread of 0.35% per month.

For the 6/12 strategy, the positive return spread is obtained for only three ranking criteria: R-ratio(0.05,0.05) with 0.58% per month, R-ratio(0.01,0.01) with 0.13% per month and R-ratio(0.3, 0.4) with 0.028% per month. While R-ratios continue to provide good performance with holding period extended to 12 months, the performance of the 6/12 strategy using cumulative return criterion deteriorates dramatically with the negative spread return of –0.21% per month. The relative performance ranking of the Sharpe ratio for 6/12 strategy compared to 6/6 strategy is unchanged with the average spread of -0.46% per month. For every criterion, none of the obtained average spreads of the 6/12 strategy exceeds the average spread of the 6/6 strategy.

[Insert Table I here]

The average spread returns for the R-ratio(0.05,0.05) and R-ratio(0.01,0.01) are positive for the 12/6 strategy, and are 0.54% and 0.50% per month, respectively. 12/6 strategy using cumulative return criterion follows the performance after the R-ratio in the third place and earns spread return of 0.23% per month. Spread returns obtained using STARR ratios are higher than that of the Sharpe ratio but lower than those of the cumulative return and the two best performing R-ratios. However, for the 12/12 strategy, the STARR(95%) ratio and the STARR(99%) ratio criteria earn the best spread returns overall of 2.18% and 2.17% per month, respectively. These results (before adjustment for transaction costs) correspond to annual momentum portfolio returns of approximately

25%, which represents an excellent performance measured by investment industry standards. Following the largest spread returns for two STARR ratios, the next largest spreads for the 12/12 strategy are obtained using the R-ratio(0.01,0.01) and R-ratio (0.,05,0.05) with 0.46% and 0.44% per month, respectively. Although cumulative return criterion for the 12/12 strategy drops in relative ranking performance compared to the 12/6 strategy and other criteria, its spread return of 0.22% per month does not differ considerably from its spread in the 12/6 strategy. Interestingly, the Sharpe ratio achieves the worst performance for 12/6 and 12/12 strategy with a spread return of –0.19% and –0.33% per month, respectively. Values of the Kolmogorov-Smirnov statistics for all criteria and strategies are between 0.0001 and 0.0717. For the majority of the criteria, the values are in the narrower range between 0.001 and 0.005.

Figure 1 presents the graph of a sample path of cumulative returns of winner and loser portfolios over all holding periods for the STARR(50%) ratio which obtains the best performance for 6/6 strategy in terms of average monthly spread. Figure 2 plots the cumulative realized profits (accumulated difference between winner and loser portfolio return over the whole period) to the 6/12 month strategy for the Sharpe ratio, STARR(99%) ratio, R-ratio(0.05,0.05) and cumulative return criterion. It is obvious that the graph of the cumulative return for Sharpe ratio criterion provides the worst performance since its sample path is dominated by sample paths of other ratios and its value of the total realized return of the portfolio at the end of observed period is lower than the total realized return of the winner-loser portfolio of each other ratio.

[Insert Figure 1 here]

[Insert Figure 2 here]

Comparing spreads of every ratio for the four strategies, the spreads to the 6/6 strategy are higher than of any other strategy, except for the two highest spread returns in 12/12 strategy using STARR ratios. The lowest spreads are obtained for the 6/12 strategy. Overall, the largest winner-loser spread is obtained on the 12/12 strategy using the

STARR(95%) ratio and the smallest on the 6/12 strategy using the same ratio. It seems that the 6/12 strategy is not as efficient in producing persistent returns as the 6/6 strategy, since the information captured by the criteria may have limited ability to capture persistence in the holding period that is longer than the ranking period. The 6/6 strategy, except the 12/12 strategy using STARR ratio(95%) and STARR ratio(99%), achieves the highest and most consistent spreads for the majority of criteria.

Comparing the results for different ratios across four strategies, it is notable that the Sharpe ratio is one of the worst ranking criteria since it obtains the lowest spread returns. It produces only one positive spread (for the 6/6 strategy). For strategies using 12-month ranking period, the Sharpe ratio produces the lowest spread return. Cumulative return criterion performs considerably better than the Sharpe ratio, since it ranks higher according to the performance based on the spread for each strategy, and generates only one negative spread (-0.02% per month for 6/12 strategy). The worst momentum spread using cumulative return criterion is obtained for the 12/12 strategy with a rank of 7. For the 6/6 and 12/6 strategy, cumulative return criterion is ranked third in performance. It appears that the more risky strategy with cumulative return criterion earns a larger spread return than a risk-adjusted strategy using the Sharpe ratio. We can reduce the riskiness of the winner-loser portfolio, for example, by not choosing the extreme decile portfolio P10 (winner) and P1 (loser), but adjacent portfolios P9 and P2 which may still produce a positive spread. If we seek risk-adjusted performance that select stocks near extreme returns, we would still be able to obtain a positive spread. So the winner stocks for the Sharpe ratio criterion would probably be selected from decile portfolios with moderate performers selected by the cumulative return criterion. R-ratios in our study deliver the most consistent performance across all four strategies. Specifically, the R-ratio(0.05, 0.05) is among the top four performers (measured by average spread) for each strategy and produces the highest spread for the 6/12 and 12/6 strategy while the R-ratio (0,01,0.01) obtains the second best spreads for the 6/12 and 12/6 strategy, respectively, and the third best performance for the 12/12 strategy. More importantly, the R-ratio obtains remarkably consistent performance across strategies with the positive winner-loser spread in the range from 0.44% to 0.77% per month. The Sharpe ratio, however, yields only one positive spread of 0.35% per month for the 6/6 strategy.

What is the reason for such superior performance of the R-ratio compared to the other ranking criteria within a specific strategy? Why does the R-ratio(0.05,0.05) obtain such a consistent performance across all four strategies? As explained in the previous section, the R-ratio is the ratio between the expected tail loss of the opposite of excess return at a given confidence level and the ETL of the excess return at another confidence level [Insert Footnote 18 here]. Therefore, the R-ratio (0.05,0.05) measures the reward at the 95 percentile threshold level and the expected value of portfolio returns so that VaR(95%) has been exceeded and most of the tail risk captured. The R-ratio with these parameters capture well the distributional behavior of the data which is usually a component of risk due to heavy tails. Both R- ratios that capture the risk of the extreme tail (covered by the measures of ETL1% and ETL5%) provide the best performance among R-ratios. STARR ratios with ETL25% and ETL50%, corresponding to R-ratio(1,0. 25) and R-ratio(1,0.50) that capture medium tail risk and entire downside risk respectively, provide better results than R-ratios with equal parameters a and ß for 6/6 strategy. However, their performance is not consistent across all strategies. The STARR(50%) ratio covers the entire downside risk and obtains the similar spread values for 12/6 and 12/12 strategies and the best performance for the 6/6 strategy (almost four times multiple than strategies with 12-month ranking period). Medium tail risk is measured by the STARR(75%) criterion and its performance measured on spreads is very similar to that of the STARR(50%) – it obtains the second best spread for the 6/6 strategy and medium spread values for three other strategies. Given the larger data set of returns for the 12/12 strategy, the STARR ratios with a higher confidence level (STARR(95%), STARR(99%)) and corresponding R-ratios with parameters 0.05 and 0.01 covering extreme tail obtain better results**.**

Table II reports the result on the final wealth of the momentum portfolios at the end of the observation period (end of the last holding period) for different criteria. Generally, the results and their relative rankings within specific momentum strategies reflect those from Table I.

For estimation of the final wealth of the momentum portfolio, we assume that the initial value of the winner and loser portfolio is equal to 1 and that the initial cumulative return CR_0 is equal 0 at the beginning of the first holding period. We then obtain the total return of the winner and loser portfolio and their difference is the final wealth of the portfolio. Given continuously compounded returns, the cumulative return CR_n in each holding period, is given by

$$CR_k = CR_{k-1} + x'_{M(k)} r_k \qquad (9)$$

where $x'_{M(k)}$ is the momentum portfolio and r is the vector of continuously compounded returns in the holding period k, $k = 1$. The total cumulative return at the end of the whole observed period is the sum of the cumulative returns of each holding period.

Final wealth for the 6/12 strategy is negative for all ratio and cumulative return criterion except for the R-ratio(0.01,0.01), R-ratio(0.05,0.05), and R-ratio(0.3,0.4). Final wealth for the 12/16 strategy is positive for most of the ratios except for the Sharpe ratio, R-ratio(0.5,0.5) and STARR(99%) ratio. However, comparing the final wealth of the 12/6 strategy with the 6/6 strategy for each ratio, the magnitude of final wealth for the 12/6 strategy is considerably lower except for the R-ratio(0.01,0.01) and the R-ratio(0,05,0.05). These two ratios obtain the largest final wealth for the 12/6 strategy with the annualized return of 5.37% and 5.78% respectively. The values of final wealth for the 12/12 strategy are similar to those of 12/6 strategy except the final wealth for STARR(99%) and STARR(95%) ratios which provide the highest values of final wealth overall, with annualized return of 23.31% and 23.40% respectively.

We also calculate the final wealth of the momentum portfolios for strategy with one-month rebalancing and 6-month and 12-month ranking period. We do not report these results here in detail due to space considerations [Insert Footnote 19 here]. For the 6/1 strategy, the values of final wealth of portfolios are lower than those of the 6/6 strategy for each ratio except for the STARR(99%) and STARR(95%) ratio that yield total realized return of 69.93% (5.83% annual return) and 109.30% (9.11% annual return) over the whole observed period, respectively. The strategy with the 12-month ranking period and monthly rebalancing shows inferior performance on final wealth values when compared to the 12/6 and the 12/12 strategy.

[Insert Table II here]

In further analysis, we focus on the 6/6 strategy since it is the most widely examined strategy in other studies. We also select three criteria for further comparison: cumulative return, Sharpe ratio, and R-ratio(0.05, 0.05) criterion, representing benchmark criterion, traditional risk-reward criterion, and alternative risk-return ratio criterion that so far provides the most consistent performance across all strategies.

When we pursue an optimized-weighted strategy, we first solve the optimization problem for $x'_{W(n)}r$ and $x'_{L(n)}r$ given by (5) and (6), using the observations of the most recent ranking period. In this case, $x'_{W(n)}$ and $x'_{L(n)}$ are optimal winner and loser portfolios and r is the vector of continuously compounded returns at day n within the observed holding period. The final wealth of the momentum portfolio after consideration of transaction costs is obtained as the difference of the final wealth before transaction costs and total transaction costs given by (8).

As shown in Table III, for equal-weighted strategy, the cumulative return and the Sharpe ratio criteria obtain the total realized returns of 107.74% and 51.85%, corresponding to annual realized returns of 8.98% and 4.32%, respectively. The R-ratio(0.05,0.05) yields the best performance with the total realized return of 111.47% or 9.29% annually. The final wealth of the momentum portfolio for the cumulative return criterion is close to that obtained by R-ratio. For the optimized-weighted strategy, the total realized return for each criterion is considerably higher, indicating the benefit of weights optimization in the winner and loser portfolios. The R-ratio obtains more than double the better performance than the Sharpe ratio and yields a total realized return of 189.41% (15.78% annually) at the end of the observed period. The results for the optimized-weighted strategy using cumulative return are excluded, since in this case we do not take into account a portfolio's risk.

[Insert Table III here]

B. Momentum Profits after accounting for Transaction Costs

The results in Table IV show the final wealth of momentum portfolios after adjustment for transaction costs for the 6/6 strategy and three criteria: cumulative return, Sharpe ratio, and R-ratio(0.05,0.05). Both the equal-weighted and optimized-weighted strategy are examined. Adjustment for transaction costs is performed at the end of the holding period, at the rebalancing points. As explained above, we employ transaction costs of 0.78% and 0.485% of the market value of the portfolio in our analysis. After consideration of proportional transaction costs, the final wealth of the momentum portfolio is reduced. For the equal-weighted strategy and the transaction costs of 0.78%, the reduction of the total realized return of the momentum portfolio is 32.97% for the cumulative return criterion and 43.42% for the R-ratio(0.05,0.05). However, for the Sharpe ratio, the impact of the transaction cost is dramatic since only a small fraction (8.39%) of the initial final wealth remains after adjustment for transaction costs.

[Insert Table IV here]

By analyzing the turnover of the 6/6 strategy within winner and loser portfolios formed on the Sharpe ratio and R-ratio (0.05, 0.05) (i.e., number of shares changed in the portfolio after rebalancing), we observe that although the average turnover is similar for both ratios (approximately 85% for the winner portfolio and slightly higher 90% for the loser portfolio), the number of changes at rebalancing points can differ considerably between the two ratios (up to 15% at the rebalancing point), which given the different levels of total realized return at the end of each holding period can lead to a difference in results.

The highest impact of transaction costs of 0.485% is on the final wealth obtained with the Sharpe ratio where almost half of the final wealth is absorbed. The impact of the same transaction costs on the final wealth of portfolio obtained by cumulative return and R-ratio(0,05,0,05) is moderate, with the reduction in final wealth of 17.34% and 27.09%, respectively. After accounting for transaction costs for the equal-weighted strategy, it seems that the strategy using the simple cumulative return obtains a slightly better performance than the strategy based on the R-ratio(0.05,0.05) with differential annual return of 0.75% for trading cost of 0.78%. The annual differential is reduced to 0.63% for transaction cost of 0.485%. The possible reason for larger impact of the transaction cost on the R-ratio(0.05,0.05) is a higher turnover in portfolios formed on the R-ratio than that of cumulative return. Figure 3 shows the final wealth of the momentum portfolios after adjusting for transaction costs at each rebalancing period for the Sharpe ratio and the R-ratio.

[Insert Figure 3 here]

Analysis of the impact of transaction costs for optimized-weight strategy and R-ratio(0.05,0.05) reveal that the impact is weaker than for the equal-weighted strategy, with the reduction of final wealth of 24.79% and 15.16% for transaction costs of 0.78% and 0.485%, respectively. However, the performance for the optimized-weight strategy using the Sharpe ratio criterion is similar to that of equal-weighted strategy with more than half of the total final wealth being erased with the transaction cost of 0.78%. Overall, even after considering transaction costs for the 6/6 strategy, the R-ratio(0.05,0.05) obtains the best performance for the adjusted final wealth over the observed period with an apparent one-quarter reduction of final wealth for the optimized-weight strategy. Our findings that trading costs have more impact on the equal-weighted strategy than on the optimized–weighted strategy are similar to those of Korajczyik and Sadka (2004), although we consider a different optimization model.

V. Performance Evaluation of Momentum Strategies based on Risk-Return Ratio Criteria

A. Performance Evaluation Measures

In addition to evaluating the performance of the ratio criteria in momentum strategy with regards to total cumulative realized return (accumulated profits or winner-loser spreads) over the observed period, we also apply an independent performance measure [Insert Footnote 20 here] to evaluate ratios on risk-adjusted performance based on an appropriate measure of risk in a uniform manner. To that purpose, we introduce a risk-adjusted independent performance measure, $E(X_t)/CVaR_{99\%}(X_t)$, in the form of the $STARR_{99\%}$ ratio, where X_t is the sequence of the daily spreads (difference between winner and loser portfolio returns over the observed period) for which we calculate the expected return in the numerator and the expected shortfall in the denominator. The ratio criterion that obtains the best risk-adjusted performance is the one that attains the highest value of the independent performance measure.

The results of the evaluation of the ratio criteria on cumulative return and independent performance measure are shown in Table V. We also include the Sharpe ratio values as the traditional investment strategy measure. The results of the 6/6 strategy are shown in Panel A of Table V. For the 6/6 strategy, the best performance value of 0.0143 for independent performance measure is obtained for the R-ratio(0.05, 0.05) with a cumulative profit of 1.1147. It is obvious that evaluation of performance using total cumulative return and risk-adjusted measures may provide different performance rankings. For example, while the STARR(50%) and STARR(75%) ratios obtain the top performance on cumulative return measure, their risk-adjusted performance is average given the values of the independent performance measure, and below average when measured by the Sharpe ratio. In contrast, the R-ratio(0.01, 0.01) and the R-ratio(0.05, 0.05) obtain the best risk-adjusted performance on the independent performance measure, but obtain only the sixth and fourth best cumulative spread respectively. In addition, we also compare the relative rankings considering the Sharpe ratio and independent performance measure. They coincide for only three criteria. Obtained values of the Sharpe ratio are generally in magnitude several times larger (three to four times) than those of the independent performance measure for every ratio.

For the 6/12 strategy, relative ranking of the descending cumulative profits and independent performance measure is identical for the first five places. The relative ranking of criteria on Sharpe ratio and independent performance measure is the same. Only the top three performers obtain positive cumulative profit and risk-adjusted measures over the whole period. These are the R-ratio(0.05, 0.05), R-ratio(0.01, 0.01) and R-ratio(0.3, 0.4) with cumulative profits of 0.7961, 0.1742 and 0.0379, and independent performance measure values of 0.01126, 0.002857 and 0.000261 respectively. For the 6/12 strategy, the R-ratio(0.05,0.05) obtains the best performance on all three performance measures.

For the 12/6 strategy, the top three performers based on independent performance measure are the R-ratio(0.01,0.01), R-ratio(0.05,0.05), and cumulative return criterion with the independent performance measure values of 0.01007, 0.00960, and 0.00201 and cumulative profits of 0.6454, 0.6936, and 0.2920, respectively. The ranking of these top three performers given independent performance measure and cumulative spread is the same. The Sharpe ratio criterion obtains negative values and the worst performance for all three performance evaluation measures in the 12/6 strategy. Similar to the 6/12 strategy, relative ranking of criteria based on independent performance measure and cumulative profit for the 12/12 strategy is identical for the top four performers.

The largest cumulative spreads across all strategies with values of 2.8122 and 2.7974 are obtained for STARR(95%) and STARR(99%) ratios in the 12/12 strategy respectively. These two criteria also provide the largest values of independent performance measure, 0.02142 and 0.01948, across all strategies. The 6/6 month strategy provides relative large cumulative spreads exceeding 1 for four criteria (STARR(50%), STARR(75%), cumulative return, and R-ratio(0,05, 0.,05). The strategies with different combinations of ranking and holding periods (6/12 and 12/6) provide considerably lower cumulative spreads than their counterparts with the same ranking and holding periods.

For the 6/12 and 12/12 strategies, relative rankings of the Sharpe ratio and independent performance measure are identical. The Sharpe ratio performance measure identifies identical winners as the independent performance measure, and that is the alternative ratio for every strategy. The discrepancy in ranking between these two risk-adjusted performance measures, and the difference in magnitude of their values implies that they capture different notions of risk and of risk-return profiles, so that given the underlying distributions of momentum portfolio spreads, they provide different performance estimates. Modeling the momentum spread in general stable non-Gaussian distribution setting could provide the answer as to what extent there is a deviation from normality present, and which measure would give more precise estimates. The values of Sharpe ratios greater than 0.0369 imply the significant t-statistics at the 1% confidence level in the full sample, and such value is obtained only once for the 6/6, 6/12, and 12/6 strategies and twice for the 12/12 strategy.

We are also interested determining to what extent the best risk-adjusted performance of some criteria on the independent performance measure consistent with the best performance on cumulative spread. Indeed, for three out of four strategies, (namely, 6/12, 12/6 and 12/12), the two top criteria performing best on the independent performance measure obtain the two largest cumulative spreads at the same time. These criteria are the STARR(95%) and the STARR(99%) ratio for the 12/12 strategy, and the R-ratio(0,01,0.01) and the R-ratio(0.05,0.05) for the 12/6 strategy and 6/12 strategy, respectively. For the 6/6 strategy, the two best criteria on independent performance measure - the R-ratio(0.05,0.05) and the R-ratio(0.01,0.01) - obtain the fourth and fifth largest spread. These results suggest that with an application of risk-adjusted criteria, we can obtain greater momentum profits than with considering the return criterion alone. Thus, we use the ratio approach to generate a balanced risk-return performance.

We also investigate strategies with 6 and 12 months ranking periods and with one-month rebalancing. The 6/1 strategy with 6 months ranking and one month rebalancing earns considerably larger spreads for each ratio criterion than the strategy with 12 month ranking and one month rebalancing. In comparison with the 6/6 strategy, the 6/1 strategy produces slightly lower spreads for every ratio criterion with the highest spread of 1.1155 for the STARR(75%) criterion. The spread results for the 12/1 strategy are similar to those of the 12/6 strategy, with the highest cumulative spread of 0.6049. The best performance for the 6/1 strategy measured on independent performance measure is obtained for the STARR(95%) ratio and in case of the 12/1 strategy for the R-ratio(0.05, 0.05).

The cumulative spreads of the 6/1 strategy with one-month gap between the ranking and holding period are very close to those of the 6/6 strategy with the highest spread of 1.0648 obtained for cumulative return criterion and the second best spread of 1.0548 for STARR(50%) ratio. Again, the best risk-adjusted performance is obtained for the Rachev ratio(0.05,0.05). We do not report these results in detail due to space considerations.

Finally, we analyze the ranking performance of cumulative return criterion and the Sharpe ratio criterion based on the independent performance measure across strategies. Cumulative return criterion obtains (medium) ranking positions 5, 4, 3, and 6 for 6/6, 6/12, 12/6 and 12/12 strategies, respectively. For the same strategies, the Sharpe ratio criterion obtains the lowest performance rankings measured by the independent performance measure: 8, 9, 10, and 10, respectively. This implies that the Sharpe ratio as the risk-adjusted criterion for a given set of data is not providing optimal risk-adjusted performance. Cumulative return criterion, although choosing the winners with the highest return, seems to choose among them a high proportion of winners that are also persistent in risk-adjusted performance. The R-ratio(0.05, 0.05) and the R-ratio(0.01, 0.01) are the two best performing ratio criteria based on the independent performance measure across all strategies except for the 12/12 strategy where the STARR(99%) ratio obtains the best risk-adjusted performance. To summarize, for all observed strategies the alternative R-ratio - in particular, the R-ratio(0.05,0.05)- obtains the best performance based on the independent performance measure and cumulative profits.

[Insert Table V here]

B. Comparison of Results with Other Studies

Raw momentum profit returns based on risk-adjusted criteria are comparable to the results of the previous studies of momentum strategies using cumulative or total return criterion and monthly data. Taking the R-ratio(0.05,0.05) as a representative alternative risk-return ratio criterion for momentum strategies, its average monthly return before transaction cost is within the range 0.44% to 0.77% and the annualized return obtained from final wealth portfolio is in the range 4.68% to 9.29%.

Considering only cumulative return criterion, the 6/6 strategy obtains the best momentum profits. Extending the set of criteria to include ratios, the 12/12 strategy obtains the best results. For the 6/6 strategy, the cumulative return, the Sharpe ratio, and the R-ratio(0.05, 0.05) criteria obtain 0.79%, 0.35%, and 0.77% average monthly return, respectively. This estimation of raw monthly profits for the 6/6 strategy and cumulative return criterion is very close to estimation of raw monthly profits in Grundy and Martin (2001) and somewhat larger than those of Griffin, Ji, and Martin (2003) [Insert Footnote 21]. In our study, we do not notice a significant difference in results for the cumulative return, the Sharpe ratio, and the R-ratio(0.05,0.05) criteria when the one-month skipping strategy (one-month gap between ranking and holding period) using daily data is taken into account. Contrary to other studies that find discrepancy in results when considering one-month gaps before ranking period and holding period, we do not observe any considerable differences in the results for the 6/6 strategy. Comparing the turnover of stocks for the strategies, we observe that based on a risk-adjusted criteria it is higher as expected, with the average fraction of remaining stocks at the rebalancing points in the range 10-15% for winner and loser portfolios.

Considering the transaction costs, our annualized results of 6.02% for cumulative return criterion are similar to those in Lesmond, Schill, and Zhou (2003), who evaluated Jegadeesh and Titman (2001) strategy with semi-annual returns of 2.20% after trading costs based on actual turnover which is comparable to our strategy. Our results are not directly comparable with those of Korajyczik and Sadka (2004) for price impact, since they use the three-factor model of Fama and French (1993) to obtain estimates of portfolio abnormal returns and Sharpe ratios as a function of the level of initial investment. Similar to Korajczik and Sadka, we find that there are alternative optimized-weighted strategies that are more profitable than the equal-weighted strategies.

V. Conclusions

In this paper, we extend the momentum trading methodology by embedding the stock ranking criterion within the common risk-return portfolio selection framework. In addition, we introduce alternative ratios that account for a risk-return profile of the individual stocks and are not restricted to the normal return distribution assumption. These ratios are based on the coherent risk measure of the expected shortfall and can be conveniently applied at the individual stock and portfolio level. Our extension proves favorable in several aspects. Methodologically, by our definition of the risk-return ratio criterion, we utilize daily data and capture the distributional properties of stock returns and their risk component at a different threshold level of the tail distribution. Furthermore, such criterion facilitates postulation of a portfolio optimization problem by using the ratio criterion as the objective function in the portfolio optimization.

Empirically, our ratios drive balanced risk-return performance according to captured risk-return profiles of observed sample of stocks and for every examined strategy produce better results than a simple cumulative return and the traditional Sharpe ratio criterion.

We evaluate and compare different risk-return ratios and cumulative return benchmark using an independent performance measure based on the expected shortfall. Our results confirm that the alternative R-ratio and the STARR ratio capture the features of the data and their distributional behavior considerably better than the classical mean-variance model embodied by the Sharpe ratio which underperforms on cumulative spread and independent performance measure. We observe that better performance of the alternative R-ratio is represented not only by better risk-adjusted performance compared to other criteria but also by its consistency and robustness across all considered strategies with a different length of ranking and holding periods. The reason behind this superior performance lies in the compliance with the coherent risk measure's ability to capture distributional features of data including the component of risk due to heavy tails, and the property of parameters to adjust for the upside reward and downside risk simultaneously.

Additionally, by applying ratio criteria, we devise an optimized-weighted strategy that optimizes weights in the winner and loser portfolios to achieve optimal performance. Optimization is performed after initial ranking of stocks in the winner and loser portfolios based on risk-adjusted ranking criteria. The optimized strategy can be extended to the whole universe of stocks with the objective of maximizing expected portfolio return, along with the simultaneous constraints on the target level of risk for the portfolio or its parts.

Finally, we incorporate transaction costs in the evaluation of investment trading style based on risk-adjusted criteria and show that for the level of transaction costs based on empirical evidence, we can still obtain profitable momentum strategies. Although risk-return ratio criteria have on average higher turnover than that of the cumulative return criterion, they still provide considerable profits after adjusting for transaction costs. We observe that transaction costs have different impact on strategies based on different criteria and that, similar to findings of Korajzcik and Sadka (2004), have a greater impact on the equal-weighted strategy than on the optimized-weighted strategy. While the transaction costs wipe almost half of the final wealth for the momentum portfolio based on the Sharpe ratio, the final wealth of the portfolio based on the alternative R-ratio is moderately reduced.

Further efforts can be pursued in two directions. First, the properties of winner and loser portfolios in the holding periods can be examined with respect to their stable distribution properties and their dependence structure in order to provide insight into the risk structure of the portfolio and its changes during the execution of the strategy. This will contribute to our understanding of rational risk-based explanations. Second, the idea of a risk-return ratio can be extended beyond investment in the extreme (winner and loser) portfolios and be applied to devise optimized-weighted strategies with investment in all stocks and with portfolio optimization models that may use ratios, expected return and expected shortfall of the portfolio as an objective function or additional constraints. In this way, we may open an avenue to an even richer set of profitable strategies.

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Footnotes

1. Mandelbrot's (1963) and Fama's (1963, 1965) observation of excess kurtosis in empirical financial return processes led them to reject the normal assumption and propose non-Gaussian stable processes as a statistical model for asset returns. Non-Gaussian stable distributions are commonly referred to as "stable Paretian" distribution due to the fact that the tails of the non-Gaussian stable density have Pareto power-type decay.
2. In mean-variance space, winners are positioned on the efficient mean-variance frontier, so that by construction they entail the highest risk Losers are simply winners in the lower (inefficient) part of the mean-variance frontier. In the observed universe of assets bounded by mean-variance frontier, there may be riskier assets with higher variance (measured by standard deviation) than those of selected winners (or losers) but they will not qualify as winner or loser based on cumulative return criterion. Similar to return ranking criterion, we might think of applying a variance ranking criterion. However, the stocks with the greatest variance do not necessarily need to be the stocks with the greatest return at the same time and would not be on the efficient frontier.
3. This choice can be further complicated by considering which risk-adjusted criteria are more useful for an investor rather than another.
4. Expected shortfall conveys the information about the expected size of a loss exceeding VaR. For example, suppose that a portfolio's risk is calculated through the simulation. For 1,000 simulations and $a = 0.95$, the portfolio's VaR would be the smallest of the 50 largest losses. The corresponding expected shortfall would be then estimated by the numerical average of these 50 largest losses.
5. There are some notational inconsistencies in the current literature on expected shortfall. Some authors distinguish between expected tail loss and CvaR, so that only in the case of continuous random variable the definition of expected shortfall coincides with that of CvaR.

6. R-ratio stands for Rachev ratio and was introduced in the context of risk-reward alternative performance measures and risk estimation in portfolio theory (Biglova, Ortobelli, Rachev, Stoyanov, 2004).

7. A plausible extension is not to restrict the percentage share of the winners and losers, and let the optimization dictate the percentage of winners and losers in the portfolio using the mixed integer programming approach.

8. Large trades exceed the number of shares the market maker is willing to trade at the quoted bid/ask prices.

9. For these turnover probabilities, Grundy and Martin (2001) estimate the level of round-trip transaction costs that would absorb the strategy's average raw and risk-adjusted returns, as well as the level of transaction costs that would render the strategy's raw and risk-adjusted after-transaction-cost returns statistically insignificant at the 5% level.

10. A limitation of the LDV model is that the underlying true return (in a frictionless market) distribution is normally distributed, an assumption that is not consistent with studies that show that the observed return distribution is non-normal. The maintained hypothesis of this approach is that arbitrageurs trade only if the value of the accumulated information exceeds the marginal cost of trading.

11. Two proportional cost models are based on quoted and effective spreads and are independent of the size of the portfolio traded. Two non-proportional cost measures reflect the fact that the price impact of trading increases in the size of the position traded.

12. Spread cost when trading stock *i* in quarter t is estimated using the volume-weighted average effective spread for all trades recorded in the ISSM database for stock *i* in quarter t. This is more appropriate than using an equally weighted spread since the mutual fund trades are relatively large and volume-weighted average places greater weight on the cost of larger trades.

13. This implies that for the 75th percentile, transaction costs per unit of invested capital is 0.01075 (0.0078 - 0.0059/2) and for the 25th percentile, transaction costs per unit of invested capital is 0.0485 (0.0078 - 0.0059/2).

14. Portfolio construction starts after the first ranking period and ends *L* months before the end of the whole sample.

15. Lesmond et al. (2003) report that momentum investor can save 23% of the cost of the short positions and 15% of the cost of the long positions by holding the positions in these stocks in the next period.

16. To obtain monthly returns, the daily returns are multiplied by 21 (assumption of 21 trading days per month)

17. The application of t-statistics requires the independent and identical distributed (iid) normal assumption for its (asymptotic) validity. However, it is well established in empirical finance that the iid normal assumption is systematically violated by asset returns in practice – returns typically exhibit skewness, excess kurtosis, (conditional) heteroscedasticity and temporal dependence. Limiting distributions for the t-statistics and the K-S-statistics are not parameter free due to the fact that parameters of the distribution have been estimated. Limiting distributions in this case can be obtained by bootstrapping, but this is beyond the scope of this paper. Here, we use the K-S-statistics only as a distance measure.

18. The ETL measures the expected value of portfolio returns given that the VaR has been exceeded. When the ETL concept is symmetrically applied to the appropriate part of the portfolio returns, a measure of portfolio reward is obtained (see Biglova, Ortobelli, Rachev and Stoyanov, 2004).

19. The results are available from the authors upon request.

20. Unlike other studies which apply one-factor or multi-factor models as performance benchmark, we give preference to direct evaluation and comparison of performance on final wealth and risk-adjusted performance measure using daily data. The portfolio alpha's values from the factor models can be considerably influenced by the overly simplistic structure of the model with much of the idiosyncratic risk left unexplained.

21. Grundy and Martin (2001) report results of 6/6 strategy with monthly rebalancing and one month skipping between ranking and holding period. Griffin, Ji, and Martin (2003) consider 6/6 strategy without one month skipping between ranking and holding period.

Figure Captions

Figure 1. Cumulative realized returns of winner and loser portfolios for a 6-month/6-month momentum strategy and STARR(50%) criterion

Figure 2: Cumulative realized momentum profits for different risk-return ratio criteria and 6-month/6-month strategy.

Figure 3: Final wealth of the momentum portfolio for the 6/6 optimized-weighted strategy and adjustment of trading costs.

Table I. Momentum Portfolio Returns

Risk-Return Ratio	Portfolio	Ranking Period			
		J = 6		J = 12	
		Holding Period		Holding Period	
		6	12	6	12
Cumulative return (Benchmark)	Winner	0.028693	0.021318	0.022820	0.021810
	Loser	0.020730	0.023394	0.020554	0.019614
	Winner-Loser	0.007963	-0.002075	0.002266	0.002195
Sharpe Ratio	Winner	0.021523	0.014775	0.017339	0.015662
	Loser	0.018070	0.019325	0.019275	0.018957
	Winner - Loser	0.003453	-0.004550	-0.001936	-0.003294
R-ratio (0.01, 0.01)	Winner	0.020433	0.018018	0.019016	0.019530
	Loser	0.015927	0.016741	0.014007	0.014965
	Winner-Loser	0.004506	0.001276	0.005008	0.004564
R-ratio (0.05, 0.05)	Winner	0.022923	0.019748	0.019890	0.019996
	Loser	0.015245	0.013915	0.014507	0.015638
	Winner-Loser	0.007678	0.005832	0.005382	0.004358
R-ratio (0.3,0.4)	Winner	0.023968	0.017882	0.019245	0.018658
	Loser	0.016638	0.017604	0.018253	0.017597
	Winner-Loser	0.007329	0.000278	0.000992	0.001061
R-ratio (0.5, 0.5)	Winner	0.022330	0.015807	0.018023	0.017145
	Loser	0.017929	0.018599	0.019518	0.019227
	Winner-Loser	0.004400	-0.002791	-0.001494	-0.002082
STARR (99%)	Winner	0.024537	0.022489	0.023663	0.030816
	Loser	0.024972	0.026881	0.024933	0.009106
	Winner-Loser	-0.000435	-0.004392	-0.001270	0,021709
STARR (95%)	Winner	0.025660	0.022549	0.024121	0.032613
	Loser	0.023773	0.027196	0.023822	0.010789
	Winner-Loser	0.001886	-0.004647	0.000299	0.021824
STARR (75%)	Winner	0.028809	0.021543	0.022654	0.022804
	Loser	0.020564	0.024415	0.021114	0.020117
	Winner-Loser	0.008244	-0.002872	0.001539	0.002686
STARR (50%)	Winner	0.028868	0.020891	0.022759	0.021785
	Loser	0.020267	0.023380	0.020554	0.019614
	Winner-Loser	0.008600	-0.002489	0.002205	0.002170

This table reports the average monthly returns (in excess of the risk-free rate and aggregated from average daily returns) for momentum portfolios based on past *J*-month risk-adjusted returns obtained using specific risk-return ratio criterion, and held for subsequent *K* months. Monthly returns are aggregated from daily returns assuming 21 trading days in month. Loser (P1) is the equally weighted portfolio of 10% of the stocks with the lowest values of criteria over the past *J*-months, and winner (P10) comprises the stocks with the highest values of criteria over the past *J*-months. The sample includes a total of 382 stocks during the period of January 1992 to December 2003.

Table II. Final Wealth of Momentum Portfolios

Risk-Return Ratio	Portfolio	Ranking Period			
		J = 6		J = 12	
		Holding Period		Holding Period	
		6	12	6	12
Cumulative return (Benchmark)	Loser	2.7440	3.1927	2.6485	2.5275
	Winner	3.8214	2.9095	2.9406	2.8104
	Winner-Loser	1.0774 (8.98%)	-0.2832 (-2.36%)	0.2920 (2.43%)	0.2829 (2.35%)
Sharpe Ratio	Loser	2.4055	2.6375	2.4838	2.4427
	Winner	2.9240	2.0165	2.2343	2.0182
	Winner - Loser	0.5185 (4.32%)	-0.6209 (-5.17%)	-0.2495 (-2.08%)	-0.4245 (-3.53%)
R-ratio (0.01, 0.01)	Loser	2.1329	2.2848	1.8050	1.9284
	Winner	2.7984	2.4591	2.4503	2.5166
	Winner-Loser	0.6654 (5.54%)	0.1742 (1.45%)	0.6453 (5.37%)	0.5882 (4.90%)
R-ratio (0.05, 0.05)	Loser	2.0428	1.8991	1.8694	2.0151
	Winner	3.1576	2.6951	2.5630	2.5767
	Winner-Loser	1.1147 (9.29%)	0.7961 (6.63%)	0.6936 (5.78%)	0.5616 (4.68%)
R-ratio (0.3,0.4)	Loser	3.2667	2.4025	2.3521	2.2676
	Winner	2.2281	2.4405	2.4799	2.4043
	Winner-Loser	1.0386 (8.66%)	0.0379 (0.31%)	0.1278 (1.06%)	0.1367 (1.14%)
R-ratio (0.5, 0.5)	Loser	3.0356	2.5383	2.5150	2.2093
	Winner	2.3896	2.1573	2.3224	2.4776
	Winner-Loser	0.6460 (5.38%)	-0.3810 (-3.18%)	-0.1925 (-1.60%)	-0.2683 (-2.23%)
STARR (99%)	Loser	3.3488	3.6687	3.2128	1.1734
	Winner	3.4082	3.0692	3.0491	3.9708
	Winner-Loser	-0.0593 (-0.49%)	-0.5994 (-4.99%)	-0.1636 (1.36%)	2.7974 (23.31%)
STARR (95%)	Loser	3.2445	3.7117	3.0696	1.3902
	Winner	3.3488	3.0774	3.1082	4.2025
	Winner-Loser	0.2574 (2.15%)	-0.6342 (-5.28%)	0.03860 (0.32%)	2.8122 (23.40%)
STARR (75%)	Loser	2.8066	3.3321	2.7207	2.5923
	Winner	3.9317	2.9401	2.9191	2.9385
	Winner-Loser	1.1251 (9.37%)	-0.3919 (-3.26%)	0.1984 (1.65%)	0.3462 (2.89%)

STARR (50%)	Loser	2.7660	2.8511	2.6485	2.5275
	Winner	3.9398	3.1908	2.9327	2.8072
	Winner-Loser	1.1737 (9.78%)	-0.3397 (-2.83%)	0.2841 (2.37%)	0.2797 (2.33%)

This table reports the final wealth for momentum portfolios at the end of observed period for different risk-return criteria. The final wealth of winner and loser portfolios is the total realized return on these portfolios at the end of observed period respectively. Loser (P1) is the equally weighted portfolio of 10% of the stocks with the lowest values of criteria over the past *J*-months, and winner (P10) comprises the stocks with the highest values of criteria over the past *J*-months. Annualized returns for the specific strategy and risk-return ratio are given in parentheses. The sample includes a total of 382 stocks during the period of January 1992 to December 2003.

Table III. Final Wealth of Momentum Portfolios for Equal-weighted and Optimized-weighted 6-month/6-month Strategy before transaction costs

Risk-Return Ratio	Portfolio	Equal-weighted strategy	Optimized-weighted strategy
Cumulative return (Benchmark)	Loser	2.7440	n.a.
	Winner	3.8214	n.a.
	Winner-Loser	1.0774 (8.97%)	n.a.
Sharpe Ratio	Loser	2.4055	2.3112
	Winner	2.9240	3.0721
	Winner-Loser	0.5185 (4.32%)	0.7608 (6.34%)
R-ratio (0.5, 0.5)	Loser	2.0428	1.5779
	Winner	3.1576	3.4720
	Winner-Loser	1.1147 (9.29%)	1.8941 (15.78%)

This table reports the final wealth of momentum portfolios for equal-weighted and optimized-weighted strategy given the cumulative return, Sharpe ratio and R-ratio(0.05,0.05) criteria. Loser (P1) is the equally weighted portfolio of 10% of the stocks with the lowest values of criteria over the past *J*-months, and winner (P10) comprises the stocks with the highest values of criteria over the past J-months. The final wealth of momentum portfolio is the difference between final wealth of winner portfolio and final wealth of loser portfolio. Annualized returns for the specific strategy and risk-return ratio are given in parentheses. The sample includes a total of 382 stocks during the period of January 1992 to December 2003.

Table IV. Final Wealth of Momentum portfolios for Equal-weighted and Optimized-weighted 6/6 Strategy after Transaction Cost Impact

Risk-Return Ratio	Portfolio Final Wealth	Stock Ranking Criteria		
		Cumulative Return	Sharpe Ratio	R-ratio (0.05, 0.05)
Equal-weighted Strategy	No transaction cost	1.0774 (8.98%)	0.5185 (4.32%)	1.1147 (9.29%)
	Transaction cost 0.78%	0.7221 (6.02%)	0.0393 (0.33%)	0.6323 (5.27%)
	Transaction cost 0.485%	0.8905 (7.42%)	0.2206 (1.84%)	0.8148 (6.79%)
Optimized-weighted Strategy	No transaction cost	n.a.	0.7608 (6.34%)	1.8941 (15.78%)
	Transaction cost 0.78%	n.a.	0.28749 (2.40%)	1.4245 (11.87%)
	Transaction cost 0.485%	n.a.	0.4687 (3.90%)	1.6069 (13.39%)

This table reports the final wealth of momentum portfolios for equal-weighted and optimized-weighted strategies after accounting for transaction costs at rebalancing points for cumulative return, Sharpe ratio, and R-ratio(0.05,0.05). Transaction cost of 0.78% is the median transaction cost for the unit of investment capital. Transaction cost of 0.485% is the 25th percentile transaction cost for the unit of investment capital. Optimized-strategy is obtained by optimizing the weights within winner and loser portfolios. The final wealth of momentum portfolio is the difference between final wealth of winner portfolio and final wealth of loser portfolio adjusted for transaction costs. Annualized returns for the specific strategy and risk-return ratio are given in parentheses. The sample includes a total of 382 stocks during the period of January 1992 to December 2003.

Table V. Performance Evaluation of Momentum Spreads generated by Risk-Return Ratio Strategies using Cumulative Spread, Sharpe ratio and Independent Performance Measure (for Equal-weighted Strategies before Transaction Costs)

Risk-Return Ratio	Cumulative spread	Sharpe Ratio	$E(X_t)/CVaR_{99\%}(X_t)$
Panel A: 6-month/6-month Strategy			
Cumulative Return	1.0774	0.027349	0.006558
Sharpe Ratio	0.5185	0.013496	0.003038
R-ratio (0.01, 0.01)	0.6654	0.030909	0.008253
R-ratio (0.05, 0.05)	1.1147	**0.050304**	**0.014339**
R-ratio (0.3, 0.4)	1.0386	0.032300	0.007265
R-ratio (0.5, 0.5)	0.6460	0.017915	0.003979
STARR (99%)	-0.0593	0.006592	-5.218129
STARR (95%)	0.2575	0.006592	0.001631
STARR (75%)	1.1251	0.027136	0.006484
STARR (50%)	**1.1738**	0.029503	0.006965
Panel B: 6-month/12-month Strategy			
Cumulative Return	-0.2832	-0.006955	-0.001753
Sharpe Ratio	-0.6209	-0.017129	-0.004135
R-ratio (0.01, 0.01)	0.1742	0.009630	0.002857
R-ratio (0.05, 0.05)	**0.7961**	**0.039596**	**0.011267**
R-ratio (0.3, 0.4)	0.0379	0.001133	0.000261
R-ratio (0.5, 0.5)	-0.3810	-0.010784	-0.002565
STARR (99%)	-0.5994	-0.018981	-0.004982
STARR (95%)	-0.6342	-0.015842	-0.004094
STARR (75%)	-0.3919	-0.009352	-0.002405
STARR (50%)	-0.3397	-0.008307	-0.002100
Risk-Return Ratio	**Cumulative spread**	**Sharpe Ratio**	$E(X_t)/CVaR_{99\%}(X_t)$
Panel C: 12-month/6-month Strategy			
Cumulative Return	0.2920	0.007787	0.002005
Sharpe Ratio	-0.2495	-0.007479	-0.001825
R-ratio (0.01, 0.01)	0.6454	**0.036747**	**0.010072**

R-ratio (0.05, 0.05)	**0.6936**	0.035336	0.009604
R-ratio (0.3, 0.4)	0.1278	0.004372	0.001032
R-ratio (0.5, 0.5)	-0.1925	-0.005823	0.001405
STARR (99%)	-0.1636	-0.005928	-0.001508
STARR (95%)	0.0386	0.001028	0.000258
STARR (75%)	0.1984	0.005083	0.001341
STARR (50%)	0.2841	0.007582	0.001952
Panel D: 12-month/12-month Strategy			
Cumulative Return	0.2829	0.008753	0.002584
Sharpe Ratio	-0.4245	-0.015735	-0.004208
R-ratio (0.01, 0.01)	0.5882	0.034889	0.009948
R-ratio (0.05, 0.05)	0.5616	0.030459	0.008543
R-ratio (0.3, 0.4)	0.1367	0.005711	0.001566
R-ratio (0.5, 0.5)	-0.2683	-0.010067	-0.002724
STARR (99%)	2.7974	**0.070928**	**0.021429**
STARR (95%)	**2.8122**	0.062130	0.019481
STARR (75%)	0.3462	0.010200	0.003074
STARR (50%)	0.2797	0.008668	0.002555
Panel E: 6-month/1-month Strategy (1-month rebalancing)			
Cumulative Return	0.8753	0.021728	0.005155
Sharpe Ratio	0.1852	0.005206	0.001165
R-ratio (0.01, 0.01)	0.4796	0.025063	0.007118
R-ratio (0.05, 0.05)	0.4194	0.020068	0.005557
R-ratio (0.3, 0.4)	0.3684	0.011472	0.002639
R-ratio (0.5, 0.5)	0.2403	0.006897	0.001552
STARR (99%)	0.6893	0.023737	0.006779
STARR (95%)	1.0939	**0.029307**	**0.007637**
STARR (75%)	**1.1155**	0.027062	0.006647
STARR (50%)	0.8958	0.022240	0.005274
Risk-Return Ratio	**Cumulative spread**	**Sharpe Ratio**	$E(X_t)/CVaR_{99\%}(X_t)$
Panel F: 12-month/1-month Strategy (1 month rebalancing)			
Cumulative Return	0.0747	0.001957	0.000491

Sharpe Ratio	-0.2781	-0.008105	-0.001860
R-ratio (0.01, 0.01)	0.4423	0.024235	0.006224
R-ratio (0.05, 0.05)	0.5812	**0.028526**	**0.007200**
R-ratio (0.3, 0.4) R-ratio (0.5, 0.5)	-0.0028 -0.3438	-0.000093 -0.010304	-0.000021 -0.002350
STARR (99%)	**0.6049**	0.019609	0.005484
STARR (95%)	0.3068	0.007957	0.002141
STARR (75%)	0.1147	0.002924	0.000756
STARR (50%)	0.0643	0.001683	0.000426
Panel G: 6-month/6-month Strategy with 1 month skipping			
Cumulative Return	**1.0648**	0.028115	0.007182
Sharpe Ratio	0.4569	0.013712	0.003265
R-ratio (0.01, 0.01)	0.6964	0.035436	0.009178
R-ratio (0.05, 0.05)	0.8944	**0.043562**	**0.012174**
R-ratio (0.3, 0.4)	0.9722	0.032456	0.007528
R-ratio (0.5, 0.5)	0.6498	0.020379	0.004824
STARR (99%)	0.5667	0.019548	0.005445
STARR (95%)	0.4509	0.012018	0.003246
STARR (50%)	1.0548	0.027750	0.007112
STARR (75%)	0.8967	0.022463	0.005709
STARR (50%)	1.0548	0.027750	0.007112

This table reports the evaluation of momentum strategies using risk-adjusted criteria and cumulative return benchmark criterion on cumulative spread, Sharpe ratio, and independent performance measure. The values in bold denote the best criterion performance for specific evaluation measure. Independent performance measure is risk-adjusted performance measure in the form of $STARR_{99\%}$ ratio. The sample includes a total of 382 stocks during the period of January 1992 to December 2003.

Figure 1

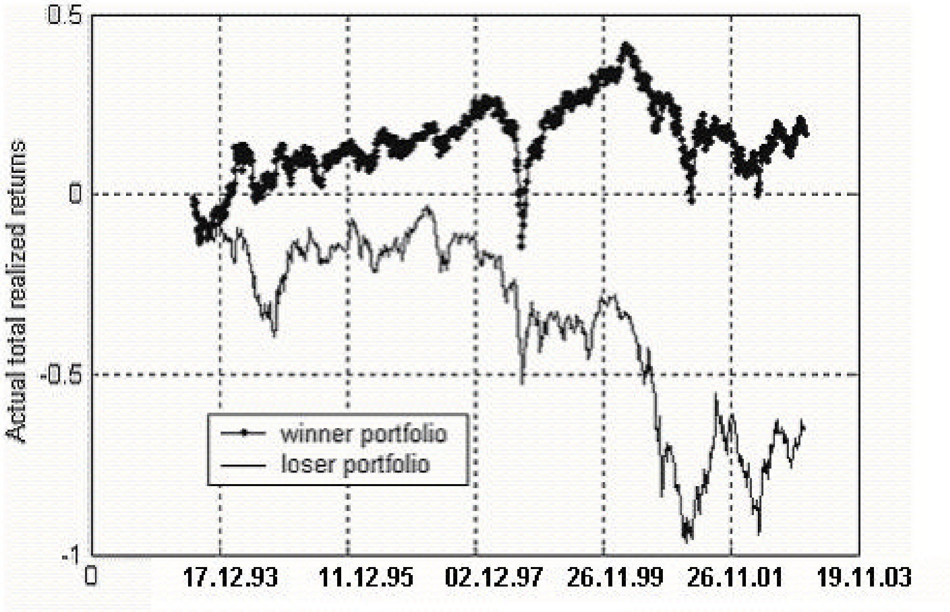

0.5
0
-0.5
-1
Actual total realized returns
winner portfolio
loser portfolio
0
17.12.93
11.12.95
02.12.97
26.11.99
26.11.01
19.11.03

Figure 2

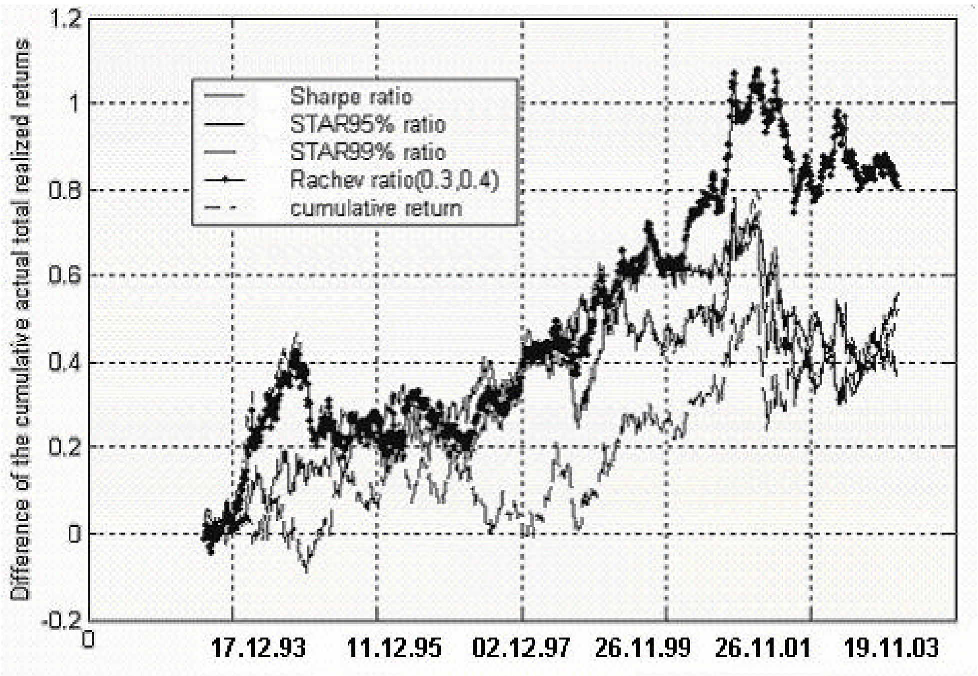
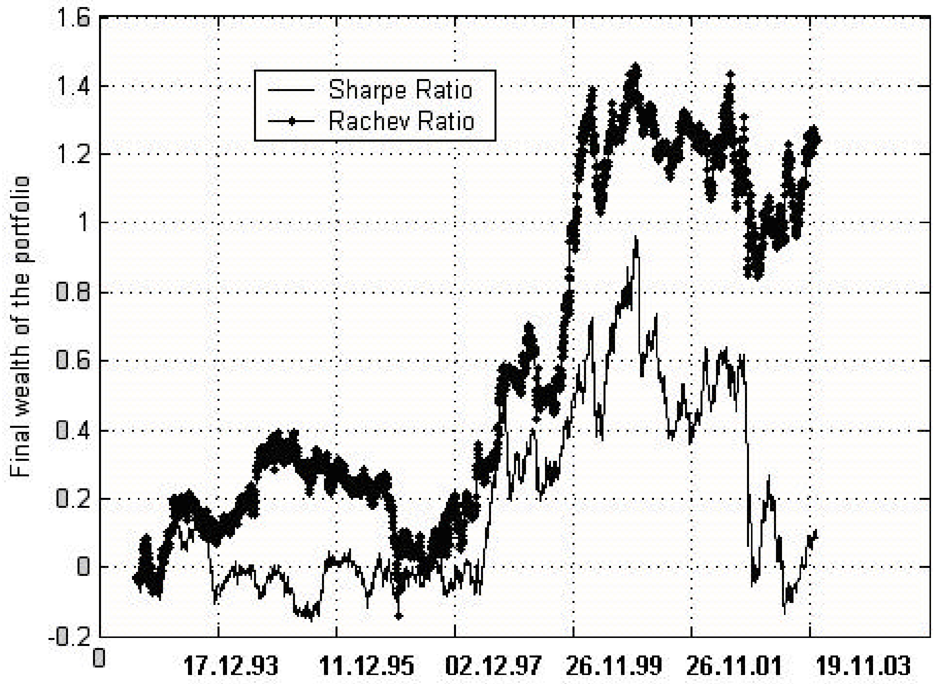

Sharpe ratio
STAR95% ratio
STAR99% ratio
Rachev ratio(0.3,0.4)
cumulative return
Difference of the cumulative actual total realized returns
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
17.12.93
11.12.95
02.12.97
26.11.99
26.11.01
19.11.03

Figure 3



Sharpe Ratio
Rachev Ratio
Final wealth of the portfolio
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
17.12.93
11.12.95
02.12.97
26.11.99
26.11.01
19.11.03