A New Hybrid Model for Intraday Spot Foreign Exchange Trading Accounting for Heavy Tails and Volatility Clustering

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Abstract

Intraday Spot Foreign Exchange market is extremely volatile and cannot be explained by macro fundamentals. Models of market microstructure have a better forecasting quality, while still cannot fully explain the exchange rates fluctuations, especially over short term and on high frequency data. In this paper, we construct a new model for explaining forex market movements on minute data. This model involves price data on two different time frames, one macro fundamental variable and accounts for volatility clustering through a GARCH approach. Alpha-stable distributions appropriately describe the behavior of residuals. The model is constructed in two variants - for market makers observing the orders flow and for traders who only have the information about the price. In both cases, the new model outperforms other previously studied models.

Keywords: Foreign Exchange Market, Intraday Trading, Heavy Tails, Volatility Clustering, Market Microstructure

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Introduction

Foreign exchange market is different from other financial markets in many respects and specific models are developed to describes its behavior. By definition, there are no insiders or particularly informed traders on forex. The exchange rate movements are theoretically defined by a relative state of two countries' economies, but macro based model still largely fail to explain short term price movements on forex. Model of market microstructure generally do slightly better, while still have low forecasting quality on high frequency data.

The present research develops a new hybrid model based on market microstructure, but also involving a macro fundamental as an explanatory variable. The model uses price data on two different time frames to account for both recent changes and longer term trends. The model uses GARCH approach to mind volatility clustering, and alpha-stable distributions explain the heavy-tailed behavior of residuals. The model is constructed in two versions, one for market makers having information about the order flow, another is for traders only observing price movements. Both models outperform previously studied macro, micro and hybrid models.

The remainder of the paper is organized as follows. The section 1 gives an overview of models for market microstructure. Section 2 describes previous attempts to construct a hybrid model. Section 3 presents and tests the new proposed model. Section 4 concludes.

1 Models of the Spot Foreign Exchange Market Microstructure

Micro-based models attempt to describe the behavior of the market, discover laws of its functioning, short-term reactions. As this analysis is based on early signs and leading indications, it is closer to technical analysis on one side and is more used for short term trades. It is for instance not uncommon to have a long term bullish trend on the market, confirmed by fundamental analysis of macroeconomic variables, while many investors and traders take advantage of short term downturns on the market, forecasted based on technical analysis and short term market movements. As a consequence, this type of analysis is the most used for intraday trading.

1.1 Orders Flow Model

Evans and Lyons [6] concentrate on the orders flow. They assume that it contains information on relevant fundamentals for two reasons:

- Traders who aim making profit on the foreign exchange market initiate trades when they believe they have information they can take advantage of, and

- Market participants who cover their other activity through forex market operations, all together, represent the current state and direction of economy.

Note that it is crucial to take into account the *signed* orders flow, rather than unsigned. Imagine a trader approaching a market maker to sell 10 lots of EUR vs. USD. Another trader comes in the same time with a request to buy 20 lots of EUR vs. USD. The unsigned order flow merely indicates that the volume of transactions reached 30 lots. The signed order flow however, will be $-10+20$, which gives $+10$ as an outcome, and provides the valuable information to the market maker that the market expects a rise of the price.

The relevant expectation (that of the market maker):

$$s_t = (1-b)\sum_{i=0}^{\infty} b^i E_t^m f_{t+i}, \tag{1}$$

where E_t^m is the expectation conditioned on market makers' information at the start of period t - this difference is crucial, as micro-based models attempt to explain the process of incorporation of available information into prices. In practice, the one estimates the model consisting in two independently estimated equations:

$$\triangle s_{t+1} = b + a x_t^{AGG} + e_{t+1} \tag{2}$$

and

$$\triangle s_{t+1} = b + \sum_{j=1}^{6} a_j x_{j,t}^{DIS} + e_{t+1}, \tag{3}$$

where

- x_t^{AGG} is aggregated order flow from six last periods of time,
- $x_{j,t}^{DIS}$ is the order flow from segment j.

The first equation estimates the forecasting power of the aggregated flow, then the second detects the input of each of the disaggregated flow individually.

Evans and Lyons [6] compare the true, ex-ante forecasting performance of a micro-based model against both a standard macro model and a random walk. The forecasting is examines for a short term period, one day to one month. Over 3 years of testing, it is shown that the micro-based model consistently outperform both random walk and the macromodel. This does not imply that past macro analysis has overlooked key fundamentals: finding consistent with exchange rate being driven by standard fundamentals.

An important reserve to be put on this models is whether the actual demand of the market is adequately reflected by the order flow. It is not uncommon to have a situation when there is an actual demand, but there is no transaction generated due to eventual limits on credit lines, trading hours or technical limitations or issues.

One should also analyze of any type of transactions are equally informative and important for the price formation. The aggregated order flow may come from many small traders each of which has its own source of information and beliefs about the market. This same net position can be generated by one transaction of an institutional client or a small bank - his source of information may be more reliable on one side, but if it is erroneous, it will generate a more important distortion on price than that serving retail clients.

Also, an important question to ask, whenever market makers know the identity of their clients, is whether orders flow generated by corporations, financial institutional and banks are more susceptible to influence the market makers perception of the market and thus the price formation.

The last remark is related to the type of orders received by the market makers. There are two types of orders: market order with immediate execution and pending orders - revocable or irrevocable, with or without expiration date, with a prespecified non market price at which the order is placed if the market hits the target price before the order expired. While it is that the first type of orders should be taken into account, the answer is less obvious for the pending orders. Should they be taken into account at all? On one side they do reflect the traders' expectation regarding future prices and even provide more details in the form of a target price. On the other hand, especially if pending orders can be revoked without financial penalty, these belief's of traders are not backed by real money and may simply reflect an attempt of an arbitrage strategy trying to catch the price "just in case". If the answer on the first question is still positive and pending orders should be taken into account, an important timing question arises - at which moment these positions have an actual influence on the market? Does it happen in the moment when the position is placed and the information about it is already available to the market maker? Or this order should rather be incorporated in the order flow in the moment of its activation when a trader commits real money into the operation? The answer on this second question has more influence if pending orders are allowed to stay active for longer time periods, eventually several days or even longer.

1.2 Evidence of Orders Flow Model on the Modern Foreign Exchange Market

To test the model described in the previous section, we take data from the foreign exchange market for the period from 1st March 2009 till 31st May 2009. The data reflects the volume of trades going through the dealing desk of a market maker. For each day of trades, there following data is analyzed:

1. Total number of lots of each currency pair bought by traders through the company,
2. Total number of lots of each currency pair sold by traders through the company,

3. Average price (quote) of the currency pair for each trading day, calculated as $\frac{Open+High+Low+Close}{4}$.

The analysis was done for seventy available currency pairs. The trades are taken from a sample with trading hours weekly from Sunday 23.00 till Friday 23.00. For the daily data, Sunday evening data was merged with Monday data, this providing 25 hours of trading information for Mondays and 23 hours of trading for Fridays.

Figure 1 shows the plots of daily exchange rate variations vs. scaled order flow of a market maker.

Several currency pairs were omitted in the experiment due to lack of regular data. We assume that the current order flow t is already known to the market maker. We also suspect the volatility clustering, i.e. current change in price depends on previous changes in price. We thus estimate the improved model 4 for the remaining currency pairs.

$$\triangle s_{t+1} = b + \sum_{j=0}^{6} a_j x_{j,t}^{DIS} + \sum_{j=0}^{6} c_j \triangle s_{t-j} + e_{t+1}, \tag{4}$$

The estimation process was iterative, whenever a coefficient was not significative, the corresponding variable was eliminated. The R^2 for the estimated models are given in the table 1 for each currency pair.

As can be observed, the orders flow model have decent explanatory power of price changes of actively traded currency pairs. It will be thus admitted as basic model for further exploration. Its step-by-step analysis is presented below.

Comparison with random walk. We want to know if the suggested model is any better than a simple random walk. We estimate the equation 5.

$$\triangle s_t = a_0 + e_t, \tag{5}$$

where $e_t \sim N(\mu, \sigma)$. The results of this calculation are have shown R^2 at zero for all the currency pairs without exception. In other words, the suggested orders flow model outperforms the random walk approximation, in average by 15.9% taking into account the currency pairs for which an appropriate orders flow model was found.

In-sample and out-of-sample data. Orders flow model estimated for the period 1st March 2009 till 31st May 2009 is now applied to out of sample daily data from 1st June 2009 till 15th June 2009. R^2 for in-sample and out-of-sample data are compared in the table 2.

As can be observed, for those cases where the model could explain the out-of-sample variations, the R^2 was well comparable for the in-sample data.

High-frequency data. As our primary purpose is to build a model for intraday trading, higher frequency data is to be analyzed. The same procedure as described above, is now applied to hourly and minute-by-minute data over the same three month period of time. For the hourly data, last 6 periods are taken into account in the model. The number of past periods is increased to 30 for the minute-by-minute data. For each model, variables with non significant

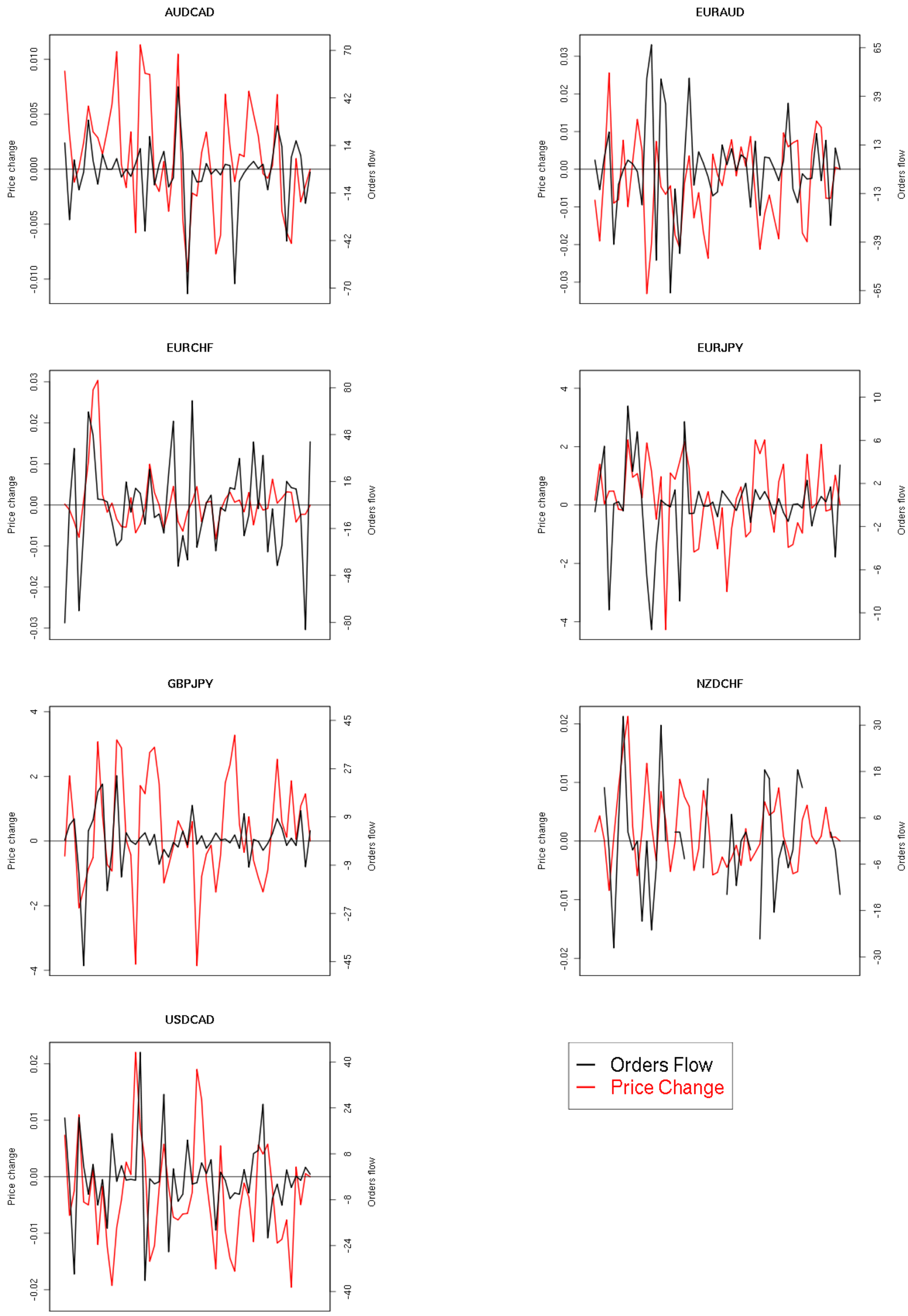


Figure 1: Daily exchange rate variations vs. orders flow.

Table 1: R^2 of the Orders Flow model estimated on daily data.

Currency pair	R^2, %	Currency pair	R^2, %
AUDCAD	20.24	EURZAR	33.87
AUDCHF	45.55	GBPAUD	35.84
AUDJPY	25.69	GBPCAD	25.89
AUDNZD	35.84	GBPCHF	43.89
AUDUSD	24.69	GBPJPY	38.24
CADCHF	29.24	GBPNZD	36.63
CADJPY	34.64	GBPUSD	45.56
CHFJPY	25.15	NOKJPY	46.96
CHFSGD	43.62	NZDCAD	29.79
EURAUD	15.95	NZDCHF	37.22
EURCAD	19.66	NZDJPY	23.39
EURCHF	45.21	NZDUSD	28.63
EURGBP	35.31	USDCAD	31.23
EURJPY	29.22	USDCHF	32.95
EURNZD	25.44	USDJPY	29.48
EURSGD	36.88	USDTRY	40.17
EURTRY	27.48	USDZAR	64.26
EURUSD	43.53		

Table 2: R^2 of the Orders Flow model for in-sample and out-of-sample data.

Currency pair	In-sample	Out-of-sample	Currency pair	In-sample	Out-of-sample
AUDCAD	20.24	32.59	EURZAR	33.87	12.13
AUDCHF	45.55	0	GBPAUD	35.84	0
AUDJPY	25.69	14.60	GBPCAD	25.89	0
AUDNZD	35.84	0	GBPCHF	43.89	34.64
AUDUSD	24.69	4.66	GBPJPY	38.24	41.32
CADCHF	29.24	0	GBPNZD	36.63	0
CADJPY	34.64	0	GBPUSD	45.56	41.93
CHFJPY	25.15	0	NOKJPY	46.96	0
CHFSGD	43.62	0	NZDCAD	29.79	3.21
EURAUD	15.95	9.18	NZDCHF	37.22	11.81
EURCAD	19.66	0	NZDJPY	23.39	0
EURCHF	45.21	0	NZDUSD	28.63	0
EURGBP	35.31	44.60	USDCAD	31.23	0
EURJPY	29.22	16.90	USDCHF	32.95	39.28
EURNZD	25.44	16.61	USDJPY	29.48	0
EURSGD	36.88	0	USDTRY	40.17	2.40
EURTRY	27.48	0	USDZAR	64.26	0
EURUSD	43.53	1.34			

Table 3: R^2 of the Orders Flow model for daily, hourly and minute-by-minute data.

Currency pair	Daily data	Hourly data	Minute data	Currency pair	Daily data	Hourly data	Minute data
AUDCAD	20.24	18.49	10.41	EURZAR	33.87	4.09	-
AUDCHF	45.55	21.91	11.56	GBPAUD	35.84	19.18	15.37
AUDJPY	25.69	20.79	19.06	GBPCAD	25.89	18.49	14.60
AUDNZD	35.84	18.27	7.21	GBPCHF	43.89	27.63	16.84
AUDUSD	24.69	20.83	15.18	GBPJPY	38.24	20.90	22.35
CADCHF	29.24	21.82	9.82	GBPNZD	36.63	22.33	-
CADJPY	34.64	22.34	16.9	GBPUSD	45.56	21.36	19.31
CHFJPY	25.15	22.71	15.88	NOKJPY	46.96	9.60	-
CHFSGD	43.62	19.89	9.96	NZDCAD	29.79	19.75	6.16
EURAUD	15.95	21.09	15.07	NZDCHF	37.22	22.08	7.57
EURCAD	19.66	19.94	12.10	NZDJPY	23.39	22.24	14.57
EURCHF	45.21	26.80	8.71	NZDUSD	28.63	22.46	9.64
EURGBP	35.31	24.80	12.93	USDCAD	31.23	22.77	11.26
EURJPY	29.22	22.49	21.38	USDCHF	32.95	43.59	10.47
EURNZD	25.44	22.54	11.18	USDJPY	29.48	22.44	11.42
EURSGD	36.88	4.98	14.29	USDTRY	40.17	19.76	-
EURTRY	27.48	24.27	-	USDZAR	64.26	4.22	1.79
EURUSD	43.53	15.24	14.40				

coefficients are iteratively removed. The calculated R^2 are shown in the table 3.

The results obtained at this step are indeed interesting. As can be observed, the coefficient of determination steadily declines as the frequency of the data increases.

Analyzing the residuals. For each equation, we now calculate the series of residuals and approximate the distribution of these series

- first by normal distribution,
- then by alpha-stable distribution.

Several residual patterns and their estimated distributions are presented on the figures 2.

To evaluate quantitatively, which distribution is more appropriate to describe the data, we will use the Integral of difference [12].

$$I = \frac{1}{2}\int_{-\infty}^{\infty} |f_{X,e}(x) - f_{X,th}(x)| dx, \tag{6}$$

where

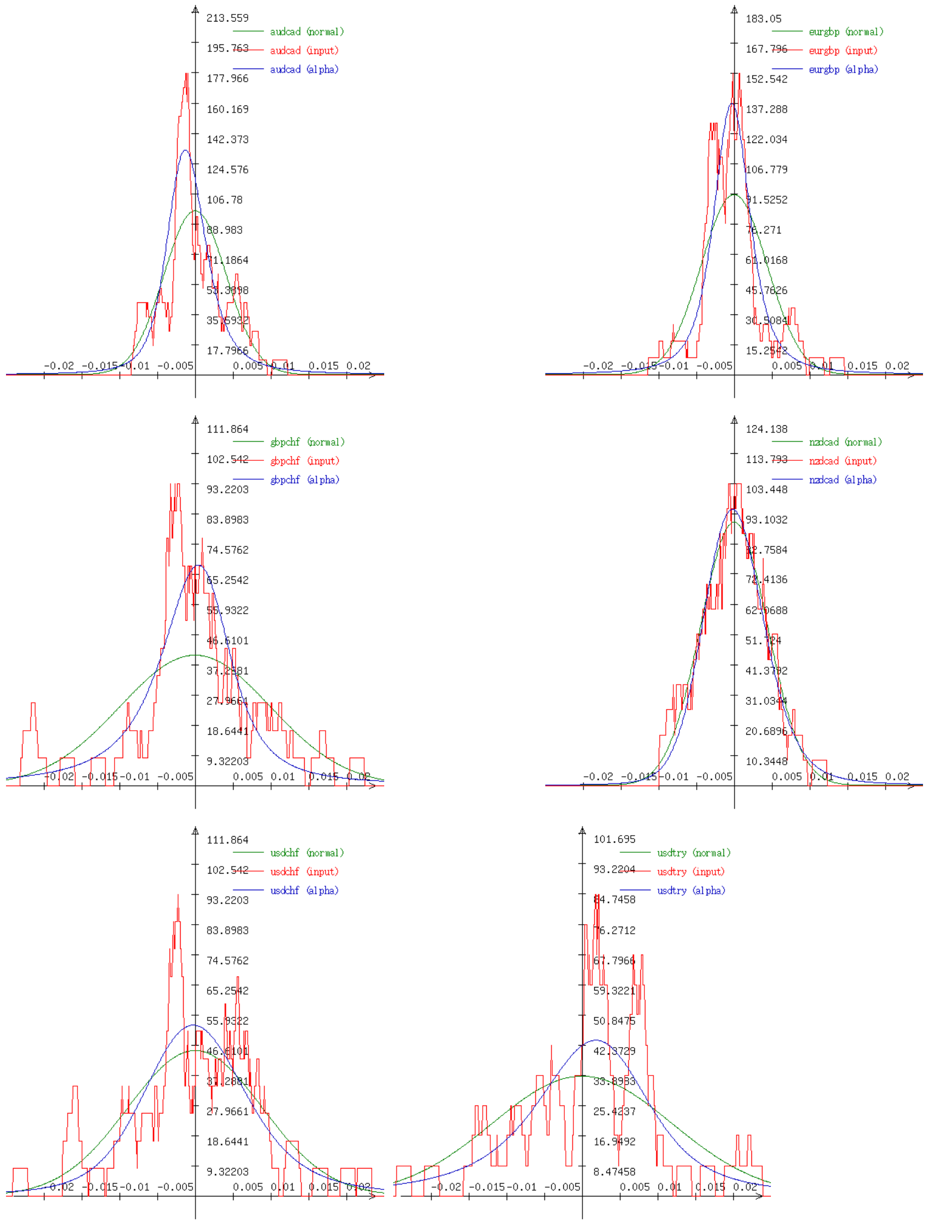


Figure 2: Empirical distribution functions (input) and their approximations with normal and alpha-stable distributions.

- $f_{X,e}$ is the empirical frequency,
- $f_{X,th}$ is the estimated, theoretical, probability distribution function.

This integral is always $0 \leq I \leq 1$ and can be interpreted as a part of the residuals' behavior, unexplained by the suggested distribution. In our particular case, we calculate

$$I_n = \frac{1}{2} \int_{-\infty}^{\infty} |f_{X,e}(x) - f_{X,normal}(x)| dx \tag{7}$$

$$I_\alpha = \frac{1}{2} \int_{-\infty}^{\infty} |f_{X,e}(x) - f_{X,\alpha-stable}(x)| dx \tag{8}$$

The table 4 shows the values the integrals 7 and 8, for each currency pair, as well as the residuals distribution selected on the basis of this calculation.

As can be observed, for all series with no exceptions, the α-stable distribution described the behavior of residuals more appropriately than the respective normal distribution.

Institutional vs. retail traders. To test whether institutional investors are better informed than individual traders, or, better to say, if their expectations are taken into account more seriously by market makers, we re-estimate the model 2 separately for the following groups of traders:

- Big traders: balance on the trading account exceeding 1 000 000 USD or equivalent in another currency,
- Medium traders: trading balance between 200 000 and 999 999 USD or equivalent,
- Retail traders: trading balance below 199 999 USD.

The R^2 for each group of clients calculated on the minute data is presented in the table 5.

Observing the results, we conclude that the predicting power of the order flow does not change depending on the financial size of traders generating the this order flow.

Instant execution and pending orders. Finally, in all the previous analysis pending orders were not taken into account at all until they become an active order, e.g. until the trader commits real money into a position, following his estimations of the market evolution. However, pending orders themselves give additional information to the market makers about the price changes the trades expect.

We re-estimate the model on the minute data and two variant of order flow calculation:

- Initially used order flow of instant execution transactions,
- Alternative order flow of both instant execution and pending orders transactions.

Table 4: Comparison of normal and α-stable residuals distribution estimation using the Integral of difference, for minute data.

Currency pair	Integral of difference		Better res. distr.
	Normal	α-stable	
AUDCAD	0.0814	0.0448	α-stable
AUDCHF	0.0952	0.0555	α-stable
AUDJPY	0.1108	0.0776	α-stable
AUDNZD	0.0874	0.0473	α-stable
AUDUSD	0.1063	0.0545	α-stable
CADCHF	0.1193	0.0519	α-stable
CADJPY	0.1222	0.0783	α-stable
CHFJPY	0.1169	0.0756	α-stable
CHFSGD	0.1206	0.0484	α-stable
EURAUD	0.1033	0.0413	α-stable
EURCAD	0.1096	0.0462	α-stable
EURCHF	0.1947	0.0468	α-stable
EURGBP	0.1399	0.0426	α-stable
EURJPY	0.1304	0.0855	α-stable
EURNZD	0.1286	0.0504	α-stable
EURSGD	0.1075	0.0472	α-stable
EURTRY	-	-	-
EURUSD	0.1481	0.0540	α-stable
EURZAR	-	-	-
GBPAUD	0.0881	0.0564	α-stable
GBPCAD	0.0986	0.0508	α-stable
GBPCHF	0.1342	0.0426	α-stable
GBPJPY	0.1367	0.0937	α-stable
GBPNZD	-	-	-
GBPUSD	0.1483	0.0626	α-stable
NOKJPY	-	-	-
NZDCAD	0.1888	0.0554	α-stable
NZDCHF	0.1338	0.0488	α-stable
NZDJPY	0.1252	0.0803	α-stable
NZDUSD	0.1476	0.0768	α-stable
USDCAD	0.1904	0.0857	α-stable
USDCHF	0.1716	0.0603	α-stable
USDJPY	0.1665	0.0740	α-stable
USDTRY	-	-	-
USDZAR	0.4074	0.2974	α-stable

Table 5: R^2 (%) estimated for big, medium and retail clients, on the minute-by-minute data. "-" means transactions data is not available.

Currency pair	Big	Medium	Retail	Currency pair	Big	Medium	Retail
AUDCAD	10.35	10.36	10.42	EURZAR	-	-	-
AUDCHF	11.55	11.55	11.56	GBPAUD	15.39	15.40	15.38
AUDJPY	19.05	19.03	19.06	GBPCAD	14.53	14.57	14.60
AUDNZD	7.24	7.19	7.21	GBPCHF	16.79	16.86	16.83
AUDUSD	15.20	15.20	15.19	GBPJPY	22.35	22.34	22.36
CADCHF	9.83	9.81	9.82	GBPNZD	-	-	-
CADJPY	16.90	16.90	16.90	GBPUSD	19.27	19.29	19.31
CHFJPY	15.89	15.89	16.89	NOKJPY	-	-	-
CHFSGD	-	9.93	9.95	NZDCAD	6.10	6.14	6.16
EURAUD	-	15.05	15.09	NZDCHF	7.51	-	7.57
EURCAD	12.10	12.07	12.10	NZDJPY	14.56	14.58	14.57
EURCHF	8.72	8.71	8.69	NZDUSD	9.63	9.71	9.62
EURGBP	12.92	12.90	12.93	USDCAD	11.26	11.29	11.26
EURJPY	21.34	21.34	12.38	USDCHF	10.56	10.45	10.46
EURNZD	11.18	11.17	11.18	USDJPY	11.48	11.43	11.42
EURSGD	-	-	14.29	USDTRY	-	-	-
EURTRY	-	-	-	USDZAR	-	1.81	1.79
EURUSD	14.17	15.09	14.40				

Using the alternative order flow, we assume that the market makers base their price estimation taking into account the pending orders. Thus the pending orders are considered when they are placed and become known to the market maker, instead of the moment real money are committed into the trading operation.

The results of the estimation using the alternative order flow gave exactly the same coefficients and coefficients of determination as the initial model using the order flow for instant execution positions only.

1.3 Other Models for Forex Market and Special Cases

After having discussed both macro and micro approach to modeling the exchange rates dynamics, it is important to note that these two model types are not incompatible. In is typically assumed for macro approach that all the relevant information is publicly known and is reflected in current market prices. If any of these assumptions is relaxed, the order flow does explain a part of the exchange rates variations.

On the other hand, the micro approach does not claim the macro fundamentals do not define exchange rates. It rather says the order flow is more dynamic and forecasts those fluctuations better. The flow of orders merely reflects the belief of market participants materialized in form of their real money put into play.

A core distinction between the two approaches is the role of trades in price determination.

Fundamentals have little to no importance for intraday trading, and that the exchange rates are too much more volatile than any fundamental. As a remedy to that situation, a hybrid model, taking into account both short term and long term variations, was proposed by [8]. The model has the following form:

$$\delta P_t = f(i, m, z) + g(X, I, Z) + \varepsilon_t, \tag{9}$$

where

- $f(i, m, z)$ is the macro component of the model,
- $g(X, I, Z)$ is the micro component of the model,
- i - nominal interest rates,
- m - money supply,
- z - other macro determinants,
- X - order flow,
- I - dealer's net positions,
- Z - other micro determinants.

1.4 Portfolio Shifts Model

Macro models are typically estimated on a monthly frequency data and have the form

$$\triangle p_t = f(\triangle i, \triangle m, ...) + \varepsilon_t, \tag{10}$$

where

- $\triangle p_t$ is the change in the log nominal exchange rate of the month,
- $\triangle i$ is the change in domestic and foreign interest rates i,
- $\triangle m$ is the change of money supply over the month,
- ε_t is the residual.

In this model, there is no place for the order flow in determining the price, any of its effects would be absorbed by the residual ε_t.

Micro approach generally leads to the following form of the model

$$\triangle p_t = g(\triangle x, \triangle I, ...) + \nu_t, \tag{11}$$

where

- $\triangle p_t$ is the rate change over two transactions,
- $\triangle x$ is the change in order flow,
- $\triangle I$ change in the net dealer position,
- ν_t is the residual.

Lyons and Evans [5] propose a new model which combines both macro and micro approach:

$$\triangle p_t = f(\triangle i, ...) + g(\triangle x, ...) + \eta_t. \tag{12}$$

The main difficulty in using this model is that the macro part of it is usually estimated based on monthly data, while the micro-part is often determined on high-frequent values - daily, hourly or even tick-by-tick. A fair and meaningful trade-off can be using daily data for both macro and micro variables, getting more frequent data for the first and aggregating the latter.

The two processes assumed in the portfolio shifts models are the following:

- As a portfolio shift occurs, it is not publicly known. It is manifested in orders on the forex market, the initial volume of which goes through market makers and then are completed by inter-dealer operations. The market learns about the shift by observing these operations.
- The shift is important enough to move the market price.

If the demand is perfectly elastic, then currencies are perfect substitutes and the Portfolio Shifts model approaches the Portfolio Balance model. But in the opposite case the portfolio shifts model is different. It has a constant asset supply and defines demand components - driven by public and non-public information. The later is reflected by portfolio shifts.

Lyons and Evans [5] estimate the Portfolio Shifts model in the following form:

$$\triangle P_t = r_t + \lambda \triangle x_t, \tag{13}$$

or

$$\triangle P_t = \beta_1 \triangle (i_t - t_t^*) + \beta_2 \triangle x_t + \eta_t \tag{14}$$

where

- $\triangle P_t$ is the change of the price between periods $t-1$ and t,
- r_t is the public information increment,
- λ is a positive constant,
- $\triangle x_t$ is the order flow,
- i_t is nominal dollar interest rate,
- i_t^* is nominal non dollar interest rate,
- β_1, β_2 are parameters,
- η_t is the residual.

Tests show that this model produces better than random-walk results for both in-sample and out-of-sample data, the forecasting being more precise over shorter period of time (39 days) rather than for a longer period of 89 days.

1.5 Evidence of the Portfolio Shifts Model

To test if the Portfolio Shifts Models is applicable to the Foreign Exchange, we estimate the following equation

$$\triangle s_{t+1} = b + \sum_{j=0}^{6} a_j \sum_{i=0}^{6} x_{j+i,t}^{DIS} + \sum_{j=0}^{6} c_j \triangle s_{t-j} + dr_t + e_{t+1}, \tag{15}$$

Overnight LIBOR rates announced daily, taken from www.dowjonesclose.com/liborrates.html, state of 29th June 2009, are taken for the values of the public information r_t. The iterative estimation was done in the same way as in the previous chapter. The table 6 allows for comparison of the R^2 between Orders Flow model and Portfolio Shifts model.

It can be observed that the addition of the macro economic variable does consistently improve the quality of modeling.

Table 6: Orders Flow model and Portfolio Shifts model estimated on daily data

Currency pair	R^2 Portfolio Shifts, %	R^2 Orders Flow, %	Currency pair	R^2 Portfolio Shifts, %	R^2 Orders Flow, %
AUDCAD	22.23	20.24	EURZAR	34.00	33.87
AUDCHF	46.17	45.55	GBPAUD	35.84	35.84
AUDJPY	26.31	25.69	GBPCAD	27.13	25.89
AUDNZD	42.67	35.84	GBPCHF	43.89	43.89
AUDUSD	25.04	24.69	GBPJPY	39.37	38.24
CADCHF	29.25	29.24	GBPNZD	39.51	36.63
CADJPY	35.05	34.64	GBPUSD	46.38	45.56
CHFJPY	25.15	25.15	NOKJPY	46.69	46.96
CHFSGD	43.79	43.62	NZDCAD	31.87	29.79
EURAUD	16.07	15.95	NZDCHF	49.10	37.22
EURCAD	19.85	19.66	NZDJPY	28.17	23.39
EURCHF	49.55	45.21	NZDUSD	31.58	28.63
EURGBP	35.44	35.31	USDCAD	31.44	31.23
EURJPY	30.43	29.22	USDCHF	33.04	32.95
EURNZD	31.84	25.44	USDJPY	29.73	29.48
EURSGD	36.90	36.88	USDTRY	42.71	40.17
EURTRY	27.95	27.48	USDZAR	75.08	64.26
EURUSD	43.73	43.53			

1.6 Hybrid Models

Any of the presented models are not necessarily and strictly used in the presented form only. They can contribute to one another. For example, Medeiros [1] suggested a hybrid model by including to the basic Evans-Lyons model [5] additional variables representing a country-risk premium. Tests performed on the Brazilian foreign exchange market showed data, showed that the model had a good R^2, which was further improved by a GARCH estimation.

2 New Model for Forex Intraday Trading

As a conclusion of the performed study, a new model is to be presented. It takes into account every test performed all over the study as a building block towards an improved model for high frequency foreign exchange modeling.

2.1 Framework

Below is the summary of main the findings made and a description of the framework for the new model.

1. **Market participants.** While taking into account the activity of all market participants, the target "users" of the developed model are active traders aiming speculative profit on the foreign exchange market, as well as brokerage companies and market makers.

2. **Trading Mechanisms.** It is assumed that no regulatory restrictions apply to trading. Transactions are done electronically by traders via brokerage companies and market makers. The electronic transmission of information is assumed to be immediate. No additional delay is present in case of trading via an intermediary broker.

3. **Trade Instructions.** Traders use immediate execution and pending orders on any available currency pair. Price for immediate execution has to be specified, i.e. there are no requests on "best possible" price.

4. **Market Efficiency.** The market is not efficient in strong and semi-strong form. Interest rate parity does not hold at all times. Carry trades are not consistently profitable. Market showed to be efficient in a weak form on minute data.

5. **Liquidity.** No major market crashes are happening. Major and small news announcements are coming regularly. The market liquidity is high and does not change depending if regional equity trading sessions is being active or not. The market liquidity does not change around the news announcements.

6. **Volatility and Risk Premia.** Volatility is appropriately measured by the Expected Tail Loss, as well as by the R-ratio. It changes over time.

Traders are risk averse. No evidence of hot potato trading was found. Volatility was found to be increased during the first and the last 30 minutes of either regional equity trading session. Foreign Exchange Market does not display fractal properties. Trading short term on higher frequency information is generally more risk than trading over long term.

7. **Trading Costs.** Trading costs are not negligible and are supposed to be incorporated in the spread. No other fee are applied.

8. **Technical Analysis.** As the market is shown to be efficient in a weak form, the technical analysis cannot provide consistently accurate prediction.

9. **Fundamental Analysis and Trading Psychology.** Fundamental analysis is susceptible to provide correct predictions. Certain industries are susceptible to move the currency exchange rates. Market prices are influenced by human behavior, in particular the overreactions.

10. **Applicability of Equity Market Models to the Foreign Exchange Market.** Equity models are generally not applicable to forex market as is, but can provide ideas and econometric tools.

11. **Macro Based Models.** Models based on macro fundamentals alone fail to explain the forex intraday market movements. The simple intuition behind is that macro fundamentals do not generally change during the day, while prices are moving permanently.

12. **Models for Forex Market Microstructure.** Models for market microstructure, in particular the model of orders flow, most appropriately explain the market behavior, comparing to other studied models:

    - Introduction of previous values of price changes (adding an $AR(n)$ part) improves the coefficient of determination.
    - Model is appropriate for the out-of-sample forecast.
    - Predicting power of the order flow does not change depending on the financial size of traders generating this order flow.
    - α-stable distribution is appropriate to model the behavior of residuals.
    - Ceteris paribus, the quality of the model decreases as the data frequency increases.
    - Addition of macro fundamentals to build a hybrid model improves the forecasting ability of a model.

2.2 New model

Taking into account the considerations above, we start the construction of the new model based on the basic order flow model. In order to account for volatility clustering, i.e. when periods of high volatility are usually followed by other periods of high volatility, GARCH approach is to be used.

Next, we have seen that the addition of a macro fundamental improves the quality of the model. We include the overnight interest rate, as it both reflects the a macro characteristic of the economy and a part of the trading costs.

The data chosen for modeling should usually match the trading time horizon. However, as it was shown, the forecasting power of a model decreases as the data frequency increases. We thus decide to include a lower frequency data into equation. From the market standpoint, this decision is confirmed by the practice, when traders usually require that data on several time frames agree on the expected direction of the market, before they engage in a trading transaction.

Technically, we observed that series showed heavy tails. To account for this data specification, we admit the residuals follow an α-stable distribution ([9], [10], [11]).

Finally, for the best fit, instead of a simple linear regression, we a looking for a more complex relationship in a spline form ([**?**], [**?**]).

$$\triangle P_{t+1} = \alpha_0 + \sum_{j=0}^{k} \alpha_{1j} \triangle P_{t-j} + \sum_{j=0}^{l} \alpha_{2j} \triangle P^{l}_{t-j} + \sum_{j=0}^{m} \alpha_{3j} r_t + \alpha_4 i + \varepsilon_t, \quad (16)$$

and

$$\sigma_t^2 = \beta_0 + \sum_{j=1}^{p} \beta_{1j} \varepsilon^2_{t-j} + \sum_{j=1}^{q} \beta_{2j} \sigma^2_{t-j}, \quad (17)$$

where

- $\triangle P_t$ price change in the moment t,
- $\triangle P^l_t$ price change in the moment t, on a lower frequency time frame,
- r_t order flow,
- i interest rates,
- α_i - parameters.

The distribution of residuals, i.e. the unexplained part of the price changes, can be approximated by an α-stable distribution. Due to extremely high market liquidity, the model will also stay valid for news announcements periods.

The purpose of the research is not only to develop a model, but also to make this model usable for everyday live trading. The order flow used in the model above is not known to most traders on the market such as speculators or hedged in interest arbitrageurs. However the order flow is positively correlated with the market liquidity, which can on its turn be reflected by the number of price ticks arriving in each particular moment of time. The model suggested for this group of market participants is the following:

$$\triangle P_{t+1} = \alpha_0 + \sum_{j=0}^{k} \alpha_{1j} \triangle P_{t-j} + \sum_{j=0}^{l} \alpha_{2j} \triangle P_{t-j}^{l} + \sum_{j=0}^{m} \alpha_{3j} v_t + \alpha_4 i + \varepsilon_t, \qquad (18)$$

and

$$\sigma_t^2 = \beta_0 + \sum_{j=1}^{p} \beta_{1j} \varepsilon_{t-j}^2 + \sum_{j=1}^{q} \beta_{2j} \sigma_{t-j}^2, \qquad (19)$$

where v_t is the number of price ticks in the period t

2.3 Evidence of the New Model

The new model is being estimated in three steps:

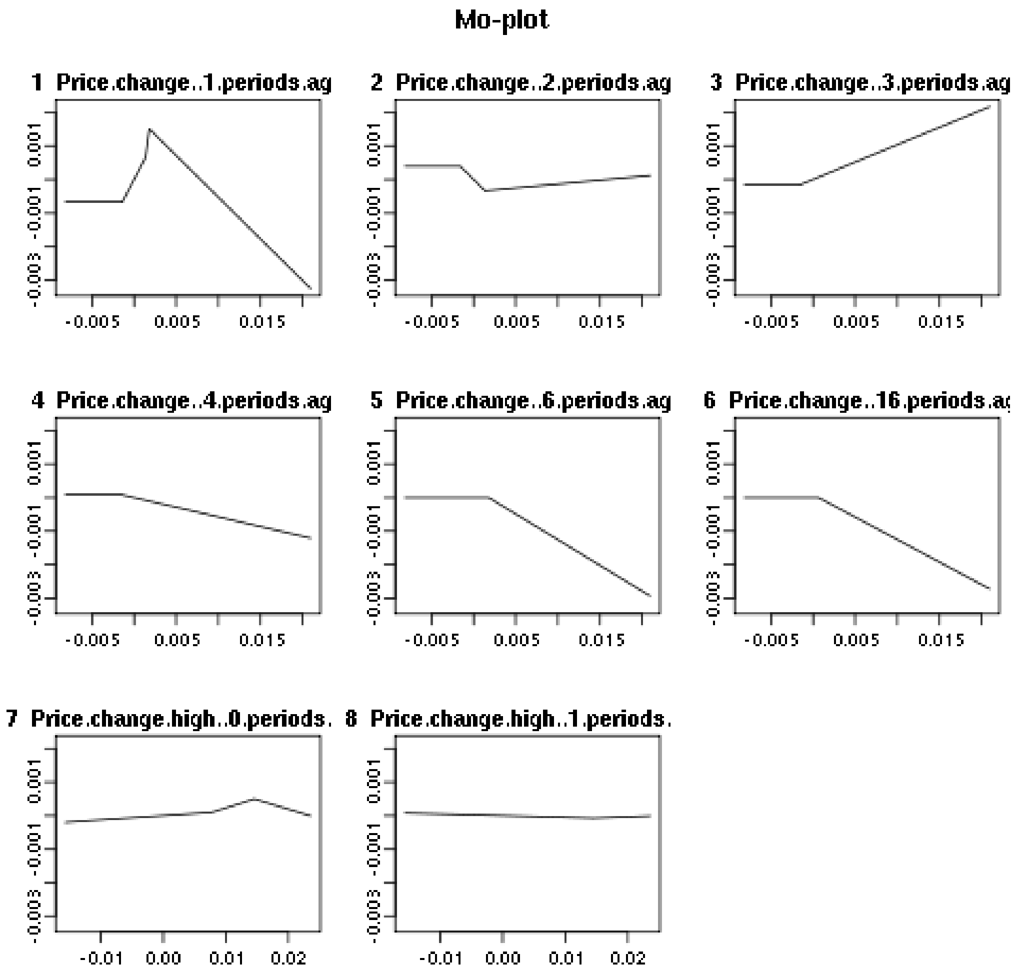
1. Apply the Multivariate Adaptive Regression Splines (MARS) to the first equation of the model.
2. Estimate the residuals using $GARCH(p, q)$.
3. Estimate the parameters of α-stable distribution of residual errors using McCulloch or any other method.

The "market makers' model" is estimated on the minute data over the last three months from 1st March 2009 till 31st May 2009. As a lower frequency time frame, hourly data is selected. To demonstrate that the model keeps the R^2 on the same level also for the out-of-sample data, tick-by-tick data between 1st and 15th of June 2009 is used. Same estimations are repeated for the "traders' model" with the same results in terms of quality of modeling.

There are three very positive results out of testing this model:

- The model for market makers having the private information about the order flow is as good as the model for traders observing the price volatility.
- This model outperforms all the previously tested models on the minute data.
- The out-of-sample performance of the model is as good as its in-sample performace.

One more observation about the model worth being mentioned here. As the equation was estimated, the MARS regression provides the analysis of impact of each independent variable on the dependent variable. Several typical charts describing this impact are presented on the figure 3.



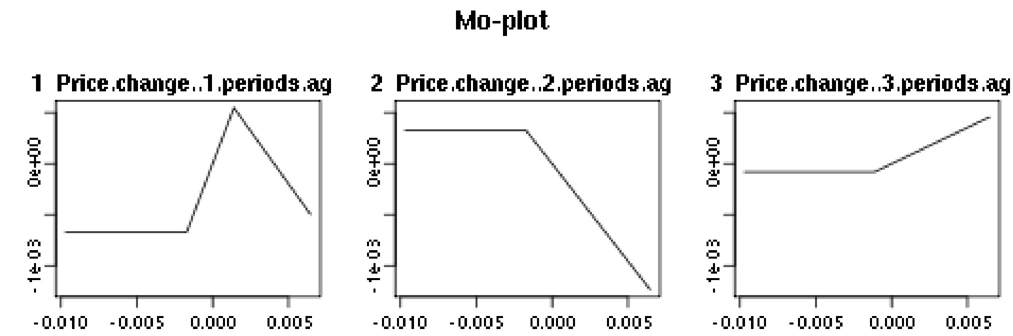


Figure 3: Influence of independent variables on the dependent variable in the new model for intraday trading.

Table 7: R^2 of the estimated new "market makers'" models, in-sample and out-of-sample data, in %.

Currency pair	R^2, in-sample	R^2, out-of-sample	Currency pair	R^2, in-sample	R^2, out-of-sample
AUDCAD	10.96	13.04	EURZAR	-	-
AUDCHF	13.48	16.97	GBPAUD	15.37	17.20
AUDJPY	20.08	20.52	GBPCAD	13.98	16.71
AUDNZD	7.59	7.81	GBPCHF	18.64	19.02
AUDUSD	16.53	17.80	GBPJPY	23.35	22.18
CADCHF	12.67	15.67	GBPNZD	-	-
CADJPY	18.28	20.59	GBPUSD	21.17	19.81
CHFJPY	17.29	18.34	NOKJPY	-	-
CHFSGD	9.48	11.97	NZDCAD	18.18	1.78
EURAUD	16.11	16.04	NZDCHF	10.39	8.69
EURCAD	14.20	15.36	NZDJPY	15.58	17.18
EURCHF	13.37	15.56	NZDUSD	10.14	11.98
EURGBP	14.83	15.32	USDCAD	14.15	18.81
EURJPY	22.48	21.95	USDCHF	15.88	22.19
EURNZD	12.10	14.23	USDJPY	15.79	8.66
EURSGD	13.70	15.82	USDTRY	-	-
EURTRY	-	-	USDZAR	64.26	
EURUSD	18.89	23.70			

As can be observed, the previous changes in price have the influence on the current change in price most of time only in their second part. In other words, increases in previous changes in price have more impact on the present change in price, than the decreases of them. This relationship has not been explored in details so far, but it is definitely another interesting research topic.

3 Concluding remarks

Step-by-step exploring and testing different aspects of the spot foreign exchange market, this research proposed a new model describing exchange rates, intended for intraday trading.

Two variants of this model were developed: one for market makers observing the order flow, one for traders who do not have this information. Both variants of the model have the same modeling quality, which is as good in-sample as out-of-sample. The R^2 of this model is higher than the R^2 of any other model tested here on the minute data.

Finally, some open questions provide room and ideas for further researched. In particular, the predicting power of the model can be further improved. Also, the observed asymetric influence in positive and negative changes in previous observed price innovations is to be explored further.

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