Problem 4

Assume portfolio value at $t=0$ is 1000$, interest rate is 0.004 per year, the stock price follows geometric Brownian motion

$$\frac{dS_t}{S_t} = 0.09 dt + 0.3 dW_t$$

Investing the money into the bank account and this stock in one year find the optimal strategy to minimize Capital-at-Risk at confidential level $\alpha=0.05$. What strategy would give us the most expected return, if we want to restrict Capital-at-Risk under 300?

Problem 5

Assume a polynomial form for the term structure of zero-coupon yields with the shape parameters:

\[ a) A_0 = 0.08, \quad A_1 = 0.02, \quad A_2 = -0.003, \quad A_3 = 0.0001 \]

\[ b) A_0 = 0.06, \quad A_1 = 0.01, \quad A_2 = -0.001, \quad A_3 = 0.0001 \]

The bond face value, annual coupon and maturity are 1000$, 5%, and 4 year. In both cases, find the price and the percentage change of the bond price, assuming the short rate increasing by 40 basic points, the slope decreasing by 10 basic points, and the changes of other parameters are zero.

Problem 6

Consider 20 observations of return of a specific portfolio 149, 25, 72, 64, 203, 14, 55, 3, 88 141, 2, 104, 135, 221, 16, 229, 245, 30, 11, 6. Suppose the return of the portfolio follows Exponential distribution with density

$$f(x) = ae^{-ax}, \text{ where } x \geq 0.$$  

*Calculate the sample AVaR$_{0.02}$.**