Homework 1

**Problem 6.6.** Assume 107, 207, 162, 61, 47, 16, −99, 269, 24, 101, 63, 173, −278, 159, 184 are 15 observations of dollar return of a specific portfolio. The dollar return of the portfolio follows normal distribution with density

\[ f(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{(x - \mu)^2}{2\sigma^2} \right). \]

Calculate the sample AVaR_{0.02}.

**Problem 6.7.** Let −12, 35, 12, −132, 373, −110, 15, 55, −13, −11, −42, −28, 283, 313, 76 be observations of return of a specific portfolio. Assume the return of the portfolio follows the Gram-Charlier distribution with density

\[ f(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{(x - \mu)^2}{2\sigma^2} \right) \left[ 1 + \frac{\xi}{6} H_3 \left( \frac{x - \mu}{\sigma} \right) + \frac{\kappa - 3}{24} H_4 \left( \frac{x - \mu}{\sigma} \right) \right], \]

where \( H_n \) is the n-th order Hermite polynomial. Calculate the sample AVaR_{0.02}.

**Problem 6.9.** Assume portfolio value at \( t = 0 \) is 1000, interest rate is 0.04 per year, the stock price follows geometric Brownian motion

\[ \frac{dS_t}{S_t} = 0.09dt + 0.3dW_t. \]

Investing the money into the bank account and this stock in one year find the optimal strategy to minimize Capital-at-Risk at confidential level \( \alpha = 0.05 \). What strategy would give us the most expected return, if we want to restrict Capital-at-Risk under 300?