Homework 1

Problem 6.6. Assume 107, 207, 162, 61, 47, 16, −99, 269, 24, 101, 63, 173, −278, 159, 184 are 15 observations of dollar return of a specific portfolio. The dollar return of the portfolio follows normal distribution with density

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\Big(- \frac{(x-a)^2}{2\sigma^2} \Big).$$

Calculate the sample $AVaR_{0.02}$.

Problem 6.7. Let −12, 35, 12, −132, 373, −110, 15, 55, −13, −11, −42, −28, 283, 313, 76 be observations of return of a specific portfolio. Assume the return of the portfolio follows the Gram-Charlier distribution with density

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\Big(- \frac{(x-a)^2}{2\sigma^2} \Big) \Big[1 + \frac{\xi}{6} H_3\Big(\frac{x-a}{\sigma} \Big) + \frac{\kappa - 3}{24} H_4\Big(\frac{x-a}{\sigma} \Big) \Big],$$

where H_n is the n-th order Hermite polynomial. Calculate the sample $AVaR_{0.02}$.

Problem 6.9. Assume portfolio value at $t = 0$ is 1000, interest rate is 0.04 per year, the stock price follows geometric Brownian motion

$$\frac{\mathrm{d}S_t}{S_t} = 0.09\mathrm{d}t + 0.3\mathrm{d}W_t.$$

Investing the money into the bank account and this stock in one year find the optimal strategy to minimize Capital-at-Risk at confidential level $\alpha = 0.05$. What strategy would give us the most expected return, if we want to restrict Capital-at-Risk under 300?