Measuring Unintended Indexing in Sector ETF Portfolios

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Note: The views expressed in this article are solely those of the authors and do not necessarily reflect the views of Credit Suisse or Finanalytica.
Abstract

We use equity sector exchange-traded funds to analyze whether and how the inclusion of several sectors in a fund of funds leads to a countering of exposures and to a replication of the index the investors are willing to diversify away from. Using various measures, we find that on a composition level, unintended indexing appears to happen in moderate degrees, but that the inclusion of too many sectors quickly leads to rather strong co-movement of simulated fund of funds with the index.

Keywords: Unintended Indexing, Portfolio Deadweight, Portfolio Maxweight, Diversification, Unintended Indexing, Exchange Traded Funds, Sectors, Multi Manager Funds, Fund Portfolios
**Introduction**

With the large and growing exchange-traded fund (ETF) industry being increasingly important to the asset management world, there is a need for research on portfolios containing ETFs. In this paper, we examine whether adding more equity sector ETFs results in nothing less than approaching the designated benchmark that the investor seeks to outperform by making sector bets. Analyzing both the compositions and exposures of the resulting portfolios built from equity sector ETFs, our study is related to discussions regarding fund portfolio or fund of fund (FoF) construction. With special problems of equity FoFs being averaging effects and dilution effects caused by holding multiple target equity mutual funds, these are not limited to portfolios built out of actively managed equity mutual funds, but may arise for equity ETF portfolios as well. In particular, the number of different funds to be included is crucial with respect to diversification and the strength of indirect exposures resulting from the weightings in a portfolio.

When building FoF portfolios, a major challenge faced by FoF managers is the selection of not only which funds or what kinds of funds to include in the portfolio, but how many funds. The challenge arises because there appears to be a trade-off between diversifying the portfolio and averaging or counter-investing. Effects of averaging out of characteristics or counter-investing may be present when diversifying holdings over too many funds, thereby involuntarily removing active bets by single managers. Examining these effects may be done by either focusing on the composition of the resulting FoFs (i.e. on the indirect exposures to stocks being held in the target equity mutual funds), or by estimating the resulting (expected) relation between the FoFs’ return and the benchmark.

In our study, we investigate the properties of portfolios built using equity sector ETFs based on the resulting (1) weight of the stocks contained in the sector ETFs and the benchmark and (2) measured return co-movements of the portfolios with the benchmark, while the benchmark here is simply the composite index containing all the companies that are included in the sector ETFs. By doing so, we are able to assess the implications regarding the index relation from both the compositions and the resulting effects of those on the constructed return series.

Increased attention in the asset management industry towards multi-asset class investments and therefore the need for transparent, clear-cut and liquid target investments has increased the use of ETF-based solutions, namely generating alpha by using beta products.
Therefore, our study is relevant not only to researchers seeking to gain insight into portfolio construction, but for portfolio managers and asset allocators as well. Naturally, the concerns of asset allocators and portfolio managers regarding the countering of exposures or increasing index-related weightings for ETF portfolios are less severe and less clear cut than when being invested in active funds. However, the basic problem that is beneath the return-to-risk discussions, i.e. the problem of tracking the indirect exposures and the limits to diversification that may arise, remains the same, only in different forms and magnitudes. With institutional money increasingly flowing into ETFs and with the emergence of beta-play in multi-asset portfolios, our study contributes to the needed analyses that should be carried out when making allocations to ETFs.

**Building FoFs – Basic Problems and Studies**

Studies regarding the building of FoFs are large in number and span the whole universe of possible fund types. We review funds of equity (mutual) fund studies in this section as these are most related to the research work presented in this paper.

In a critique of composition-related FoF building, Connelly (1997) introduces what he calls the “law of unintended indexing” that leads to “portfolio deadweight” when investing in several equity fund management styles. He argues that mixing equity fund managers, and therefore exposures, at least partially offsets bets by active managers. As a result, the portfolio holdings’ deadweight increases. Although Connelly’s critique focussed on active bets versus passive investments, in this paper we analyze whether these effects are present in portfolios containing purely passive managed funds, namely (sub-) index ETFs. We review the measure he introduced and extend it later in this study.

Using holdings level data for U.S. equity mutual funds and institutional fund, Gallagher and Gardner (2006) analyze active bet erosions in multi-manager portfolios. Their holdings-based analysis suggests significant countering in active bets as well as implications of inefficiencies in blended portfolios due to in-group trading between similar funds and their peers.

Although there are some studies that focus on fund composition and their effects when being held within a fund portfolio, like the ones mentioned above, the majority of research on FoF construction and on determining the “optimal” number of target fund to include in a fund portfolio use (simulated) return analyses. There have been several studies
that have investigated the optimal number of included funds to be included in a FoF portfolio, as this choice is crucial with respect to the possibilities of a FoF diversifying away from the designated benchmark and, ultimately, to beat it.

Louton and Saraoglu (2006), O’Neal (1997), and Fant and O’Neal (1999) investigate whether and how much different managers should be allocated when constructing diversified portfolios. Analyzing different types of mutual funds (equity, bond, balanced, and money market) and simulating possible terminal wealth outcomes for the respective categories and asset classes, Louton and Saraoglu (2006) find a significant reduction in terminal wealth variation when investing in about six funds within a fund investment objective category. However, they do not find superior performance versus benchmarks when holding multiple funds. The reduction in terminal wealth variability that they observe roughly mirrors the findings of O’Neal (1997) and Fant and O’Neal (1999).

Examining Australian equity funds, Brands and Gallagher (2005) find that only six funds are needed to significantly reduce the variability; that is, holding more than six funds, results in only a minor additional benefit, Louton and Saraoglu (2008), however, find that there are benefits with more than six funds, analyzing portfolios that include several investment objectives.

Yet despite the common findings in the reduction in risk or variability of returns, the performance or relative performance analyses results are mixed. This is consistent with the findings of Stein and Rachev (2010) who, although reporting significant reduction in time-series variability of returns for six to eight equity funds, find underperformance of style-neutral FoFs versus the benchmark.

In general, the findings of underperformance versus benchmarks by Louton and Saraoglu (2006) and Stein and Rachev (2010), for example, provide justification for the critiques by Connelly (1997) — whose deadweight argument is used and extended in our study — and diBartolomeo (1999), who stresses the danger of underperformance in decentralized multi-manager portfolio solutions.

From our brief review of the literature, it is obvious that portfolios constructed from actively managed mutual funds appear to remove both extreme outcomes as well as specific characteristics of funds included. However, while most of those studies focus on the possible return outcomes through simulation analyses using reported time series of returns, the
respective holdings of the funds composing FoFs, and therefore the resulting simulated
overall exposures, were not analyzed.

**Portfolio Deadweight and Portfolio Maxweight**

Connelly (1997) introduces a measure that he refers to as the “portfolio deadweight score” for
any equity fund as the minimum portion of all assets to which one would have exposure when
investing in the fund or the index. Put another way, portfolio deadweight is defined as the
sum of the minima of each company’s weight in either the index or the fund under
consideration:

\[ dw = \sum_{i=1}^{n} \min(w_{i,\text{index}}, w_{i,\text{fund}}) \]

We show Connelly’s original example in Table 1, augmented with one of our own.
From the examples it is obvious, that only the positions which the fund management is
underweighting against the index reduce portfolio deadweight.

<table>
<thead>
<tr>
<th>Example 1</th>
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<th>Example 2</th>
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<tbody>
<tr>
<td></td>
<td>Asset</td>
<td>Index</td>
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<td></td>
<td>1</td>
<td>10</td>
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<td>10</td>
<td>10</td>
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<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
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</tbody>
</table>

*Table 1.* Connelly (1997) and own example for portfolio deadweight

Connelly (1997) in his critique states that because FoFs invest in mutual funds that
have different styles and therefore bets against the index, FoF investors may end up investing
in a costly index product. Using Connelly’s example (i.e., example 1 in Table 1), such a
situation may arise if a second fund is underweighted by 1% in positions 1 to 5 and
overweighted by 1% in positions 6 to 10. An equal monetary allocation to these two funds
would result in an indirect exposure of exactly 10% for each position, increasing the FoF deadweight to 100%. This is what Connelly labels the “law of unintended indexing”.

We propose another weight measure in our study, the measure of portfolio maxweight, defined as:

\[ mw = \sum_{i=1}^{n} \max(w_{i,\text{index}}, w_{i,\text{fund}}) - 100 \]

We subtracted 100 from the measure in order to have the same bounds of 0 and 100 just as for the deadweight measure. Using in Table 2 the same examples as for the deadweight score in Table 1, it is obvious that each portfolio’s maxweight score is increasing only through the positions which the fund is overweighted against the index. The portfolio maxweight score may be interpreted as a measure of the strength of overweighting that results from the allocations made.

<table>
<thead>
<tr>
<th>Example 1</th>
<th>Example 2</th>
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<tbody>
<tr>
<td>Asset</td>
<td>Index</td>
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<td>1</td>
<td>10</td>
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<td>2</td>
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<td>4</td>
<td>10</td>
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<td>5</td>
<td>10</td>
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<td>6</td>
<td>10</td>
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<td>7</td>
<td>10</td>
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<td>8</td>
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<td>9</td>
<td>10</td>
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<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>100</td>
</tr>
</tbody>
</table>

Table 2. Portfolio maxweight with examples from Table 1

**Methodology**

With an understanding of portfolio deadweight and portfolio maxweight, we now explain our methodology. In this study where we use sector or sub-index ETFs, we have no underweighting or overweighting by fund managers. However, each sector ETF has a different composition because only the companies of the specific industries are included in the respective ETFs. Therefore, when building portfolios using sector ETFs, we may expect that by increasing the number of sectors included in the portfolio, deadweight increases,
gradually approaching an index value of 100%. Conversely, the maxweight score is expected to decline as the numbers of sectors included in the portfolio increases. However, a priori we do not know the severity of the effects. Consequently, to investigate these effects empirically, several different portfolios consisting of the sector ETFs must be constructed and compared to the index.

In the following, each portfolio’s weight scores are being defined as the sum of the minima/maxima of all assets included in the portfolio via allocation to the sector ETFs with respect to the index. The equal weighted portfolios are easily built by constructing weighting schemes for differing portfolio sizes. So first the number of different sectors to be included is chosen, followed by randomly selecting the sectors to be included. We use 1,000 portfolios for each size. Of course, the 1,000 portfolios for the maximum number of ETFs to be included would all be the same when every ETF is contained. On the other side one would end up with approximately 1,000 divided by the number of total ETFs different portfolios.

With the purpose of the weight measures being to gain insight on how the inclusion of sectors affects the possibilities of having allocations that do not resemble the index, it is worthwhile comparing the composition-focussed weight measures to numbers that are informative about the possible outcomes generated by the weightings.

Before we turn to the co-movements between the portfolios and the index, we take an intermediate step by introducing two measures that are related to the weight measures, but that take into account the market co-movement of each stock included in the portfolios. Accordingly, we introduce the beta scores of the weightings, referred to as minbeta and maxbeta, which are defined as follows:

\[
\min\beta = \sum_{j=1}^{n} \min(w_{i,index} \beta_i, w_{i,fund} \beta_i)
\]

\[
\max\beta = \sum_{j=1}^{n} \max(w_{i,index} \beta_i, w_{i,fund} \beta_i)
\]

One needs to run a regression of each company’s returns on the index returns and then weight the betas with the portfolio weights and the index weights in order to compare the weighed betas. This may be accomplished by a simple linear regression model:

\[
r_i = \alpha_i + \beta_i r_{index} + \epsilon_i
\]
Accordingly, with minbeta being the sum of the minima of all the products of portfolio weight and beta, this beta score corresponds to the deadweight score. The same holds true for maxbeta that corresponds to maxweight. This is shown in Table 3 using the Connelly (1997) example.

<table>
<thead>
<tr>
<th>Asset</th>
<th>Index</th>
<th>Fund</th>
<th>BW</th>
<th>MW</th>
<th>Beta</th>
<th>Weighed Beta</th>
<th>Weighed Fund Beta</th>
<th>Minbeta</th>
<th>Maxbeta</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>10</td>
<td>11</td>
<td>10</td>
<td>11</td>
<td>1</td>
<td>0.1*1=0.1</td>
<td>0.11*1=0.11</td>
<td>0.1</td>
<td>0.11</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>11</td>
<td>10</td>
<td>11</td>
<td>1</td>
<td>0.1*1=0.1</td>
<td>0.11*1=0.11</td>
<td>0.1</td>
<td>0.11</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>11</td>
<td>10</td>
<td>11</td>
<td>1</td>
<td>0.1*1=0.1</td>
<td>0.11*1=0.11</td>
<td>0.1</td>
<td>0.11</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>11</td>
<td>10</td>
<td>11</td>
<td>1</td>
<td>0.1*1=0.1</td>
<td>0.11*1=0.11</td>
<td>0.1</td>
<td>0.11</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>11</td>
<td>10</td>
<td>11</td>
<td>0.9</td>
<td>0.1*0.9=0.09</td>
<td>0.11*0.9=0.099</td>
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<tr>
<td>6</td>
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<td>9</td>
<td>10</td>
<td>0.9</td>
<td>0.1*0.9=0.09</td>
<td>0.09*0.9=0.081</td>
<td>0.081</td>
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<td>7</td>
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<td>9</td>
<td>9</td>
<td>10</td>
<td>0.9</td>
<td>0.1*0.9=0.09</td>
<td>0.09*0.9=0.081</td>
<td>0.081</td>
<td>0.099</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>9</td>
<td>9</td>
<td>10</td>
<td>1.1</td>
<td>0.1*1.1=0.11</td>
<td>0.09*1.1=0.099</td>
<td>0.099</td>
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<td>9</td>
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<td>10</td>
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<td>0.1*1.1=0.11</td>
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<td>10</td>
<td>1.1</td>
<td>0.1*1.1=0.11</td>
<td>0.09*1.1=0.099</td>
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<td>0.11</td>
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<td>5</td>
<td></td>
<td></td>
<td></td>
<td>0.949</td>
<td>1.049</td>
</tr>
</tbody>
</table>

Table 3. Portfolio beta scores using the Connelly deadweight example weights

However, the measures that sum up the weighted beta exposures do not take into account correlations and therefore are only indicative of the cumulated separate exposures. Consequently, for all simulated FoF return time series we therefore run the regression on the index returns, and calculate the beta of the portfolio with the index:

\[ r_{fof} = \alpha_{fof} + \beta_{fof} r_{index} + \epsilon_{fof} \]

From the analysis of the betas, we can see how the weight scores are linked to measurable exposure, especially whether as one would expect, the beta of any FoF will tend towards unity for increasing deadweight and decreasing maxweight. We test the one-year betas using daily data. Although the time-horizon is considerably short, we believe our selection of one year is appropriate because it is sufficient to see how composition and co-movement with markets are linked. In addition, the ETF compositions are based on the current date, and any further backtracking would mean that the weightings were even further away at the beginning of the time series than for the one-year distance.
Data – ETFs on Sectors and the Index

We use iShares ETFs for our composition-based analysis. The index used is the ETF on the STOXX Europe 600 index; the sectors in this index are represented by the respective 18 sector ETFs that track the performance of the sector sub-indices of the STOXX Europe 600 Indices.iv

Our examination indicates that the ETFs track the performance of the (sub) indices very closely, making the analysis possible using the ETFs rather than the indices themselves.v At least, this is practicable as well in the sense that the ETF products are directly investable with less effort and cost than buying the basket of equities. Accordingly, the use of ETFs is straightforward for a fund portfolio analysis, with ETFs representing the index exposure that can be directly accessed by FoF managers. Both compositions (October 31, 2010) and return data (November 1, 2009 until end of October 2010) are publicly available from the iShares homepage.

Empirical Results

Weight Score Results

As described, we calculate portfolio deadweight and maxweight scores using equally weighted portfolios for portfolio sizes containing up to 18 sector ETFs. We show whether and how the building of the sector ETF portfolios affects the FoF weight score measures in Figures 1 and 2, where the results are sorted for the sake of brevity.

From the plots of the deadweight scores for each number of included sectors shown in Figure 1, as expected we can see that the range of the deadweight to be incurred is very large. It is obvious that for an increasing number of sector ETFs included in the equally weighted portfolios, the deadweight scores base value increases. This means that by introducing more and more sectors, all others things unchanged, deadweight tends to increase. In addition, one can see that (1) the curves are getting flatter and the variation in deadweight within a given number of included sectors decreases with an increasing number of sectors and (2) the distance between the lines decreases with an increasing number of sectors included.
Our results have a straightforward implication when it comes to sector inclusions in portfolios or mandates. First, the more sectors included, the higher the deadweight. Second, the more sectors already included, the higher the minimum deadweight to be incurred but the less the effect of any additional sector on the deadweight score.

It is interesting to see that even 80% deadweight is never reached, no matter which sectors are combined. This is due to the fact that the companies representing the sub-indices have weightings versus each other within the sub-indices that are different from those that constitute the index as the sector ETFs differ in terms of numbers of companies included. Furthermore, not all sectors are represented in equal strength in the index. Based on this and the fact that we are always using equal weights, the indirect exposures to the companies in the constructed portfolios never match exactly the index weights.

Based on the weight score measures, this is good news for investors willing to diversify away from the index as even when being invested in a large number of sectors, the portfolio does not result in pure index replication. So the law of unintended indexing appears not to be so strong that 100% deadweight may be reached. In addition, one could expect that when asset allocation on a sector investment basis is undertaken, this can be accomplished with only some sectors rather than with almost all of them, so equal weight sector portfolios may apparently be built without having severe effects of unintended indexing.

The implications from the deadweight scores are strengthened when looking at the maxweight scores, as the slopes of the curves and the distances between them decrease with increasing number of funds, as does the maxweight score itself. Even for half of all sectors included in the portfolio, the score may remain between 40% and 60%, implying that the weightings still are away from the index in significant manner.

Histograms of the deadweight and maxweight scores for all possible numbers of ETFs to be included in the portfolio are shown in Figures 3 and 4, respectively. These figures support the findings reported above because the mean and the median move towards higher deadweight scores (lower maxweight scores) with increasing numbers of sectors included. It is obvious as well that the minimum deadweight that must be incurred is rising and that the highest maxweight score that may be reached is decreasing no matter the combination of sectors for larger numbers of sectors included.

-Figures 3 and 4 about here –
Beta Score Results

Following the weight score analyses, we continued the study by analyzing the beta scores. Doing so, we ran for each of the 600 stocks comprising the index and sector ETFs a regression on the index returns. Following this, we multiplied the betas obtained from the regression for each company stock with the weights of the simulated equal weight portfolios of all sizes and did so for the index with the index weights as well. As described, we define the beta scores as the sum of the minima and maxima of the betas in either the portfolio or the index. Figures 5 and 6 show the minbeta and maxbeta scores.

-Figures 5 and 6 about here –

The plots show the common picture of more diverse and more index-tending behaviour of portfolios with larger numbers of sectors included in the portfolio. But again we could state that from these scores, one would not conclude that the law of unintended indexing appears to be very strong, i.e. the mixing of several sectors would not lead too quickly to an index-resembling portfolio at already low numbers of sectors included. Histograms of the minbeta and maxbeta scores are shown in Figures 7 and 8. The histograms strengthen the results from above, with the distributions narrowing and the extremes tending closer to the mean, but the beta scores do not follow a strong tendency towards unity.

-Figures 7 and 8 about here –

Portfolios’ Index Beta Estimation Results

The analysis on a composition basis is interesting and yields a considerably surprising result of only mediocre deadweight scores even when investing in a large number of sectors and sort of appealing maxweight scores to achieve even with half or more of all sectors available. However, for a complete picture it is necessary to analyze the possible outcomes of the portfolios with respect to performance and index relation. Specifically, it is interesting to see whether the weight scores and the beta scores as well tend to underestimate the forces that may lead to unintended indexing. For this reason, we performed additional analysis by estimating regressions of the simulated FoF return time series on the index returns, with beta being the coefficient of the index returns and therefore the indicator of co-movement with the index for the respective sector portfolios.
Figure 9 shows the betas for each number of sectors and all simulations, sorted from smallest to highest beta obtained from all 1,000 simulations of the respective 18 different numbers of sectors to be included. As can be seen, the vast majority of portfolio is in the 0.8 to 1.2 range, indicating that the sector FoFs are increasingly dependent on the returns of the index. Put another way, the moderate degree of deadweight translates into a considerably higher dependency on market movements than expected.

This becomes clearer when looking at Figure 10, where the distributions of market betas are centred around unity rather quickly as the numbers of sector ETFs included increases. The percentage of simulated portfolios that have betas between 0.8 and 1.2 exceeds 99% (90%) for portfolios including 7 (4) or more sectors. Betas in the range of 0.9 and 1.1 are seen for 99% (90%) of the simulated FoFs that include 13 (10) sectors. Of all simulated FoFs for the 18 possible different numbers of sectors included with 1,000 simulations each, 93.3% have a beta in the range of 0.8 and 1.2, and 79.9% have betas in the even narrower range of 0.9 and 1.1. Omitting the simulations with only one sector included changes the percentages to 96.5% and 84.6%. Clearly, this shows the limits of diversification beyond the pure composition-based analyses, with FoFs increasingly producing index-related returns when more sectors are added.

Having analyzed the betas of all simulated portfolios, a look at the coefficient of determination of the regressions strengthens our findings that the weight measures, while being informative regarding portfolio compositions, seems to slightly understate the effects of unintended indexing. From Figure 11 it can be seen that only portfolios with 5 or less different sector ETFs have values below 90%; therefore, the constant and the index returns explain most of the variation in the FoF returns of the simulated portfolios.\textsuperscript{vi}

\textbf{Linking Scores and Beta Estimation Results}

From the analyses so far we can state that while seeing a clear but not very strong unintended indexing tendency from the weight and beta scores analyses at already small numbers of
sectors included, co-movements between the sector ETF portfolios and the index appear to be strong already at low numbers of sectors. This shows up as well when interpreting the scatter plots between the portfolio deadweight score and the portfolio index beta in Figure 12, and the portfolio minbeta score and the portfolio index beta in Figure 13, as the cloud of combinations is narrowing with increasing number of sectors included. Interestingly, we can see that the scatter plots in Figure 13 are almost the same as the corresponding ones in Figure 12, with the difference being a slightly positive relation with a radial shape of the scatter plots, stemming from the multiplication of the weights with the betas for each company contained.

- Figures 12 and 13 about here -

**Conclusion and Outlook**

We use existing measures like the deadweight score and introduce expansions to it with the maxweight score and the minbeta and maxbeta scores. In addition, we use standard beta estimation of simulated FoFs with the index and obtain different results than when focusing on the score measures.

All analyses are carried out by generating simulated FoFs containing sector ETFs in equal weights for differing numbers of sectors included. While the weight score measures are informative on the resulting compositions of the FoFs in comparison with the index, the beta score measures take into account the co-movement of the respective constituents of the sector ETFs with the benchmark. Having in common that both weight and beta score measures are sums of minima or maxima of contained (indirect) weights or the “beta-adjusted” weights of the companies in the index or the simulated FoFs, it appears that they tend to give implications of only moderate degrees of unintended indexing.

This may lead to an underestimation of the co-movement between FoFs and the index which is resulting from the fact that the separate company weights and exposures are summed up, neglecting the correlations between them that are crucial when determining the behaviour of the FoF on the aggregated level, rather than on the single component level. We therefore analyzed not only the score measures, but calculated for each simulated FoF the beta with the index on the aggregate level. It can be seen that indeed the weight and beta score measures are not fully sufficient in determining the degree of unintended indexing and the countering of exposures, as the betas of the simulated FoFs tend to be closely around unity at already small numbers of sectors ETFs included.
Putting the results of the weight and beta scores and the analysis of the co-movement between FoFs and the index together, we can stress that while the so-called law of unintended indexing may not be as strong as expected when using equal-weighted FoF with different numbers of sectors included, but the co-movement between the FoFs and the market appears to be strong at already small numbers of sectors. Therefore, practitioners may be best advised to carefully select their target number of sectors and especially focus on few sectors to be included, rather than investing in too many sectors and so eliminating the benefits of diversification and their chosen sector calls.
Figure 1. Sorted Portfolio Deadweight Scores

Figure 2: Sorted Portfolio Maxweight Scores
Figure 3. Histograms of Portfolio Deadweight Scores

Note: Increasing numbers of sectors from left to right and top to bottom.

Figure 4. Histograms of Portfolio Maxweight Scores

Note: Increasing numbers of sectors from left to right and top to bottom.
**Figure 5.** Sorted Portfolio Minbeta Scores

*Note: The most dispersed and lower patterns are obtained for smaller numbers of sectors included, i.e. the higher the number of ETFs in the portfolio the flatter and higher the curves and vice versa.*

**Figure 6: Sorted Portfolio Maxbeta Scores**

*Note: The most dispersed and higher patterns are obtained for smaller numbers of sectors included, i.e. the higher the number of ETFs in the portfolio the flatter and lower the curves and vice versa.*
Figure 7: Histograms of Portfolio Minbeta Scores

Note: Increasing numbers of sectors from left to right and top to bottom.

Figure 8: Histograms of Portfolio Maxbeta Scores

Note: Increasing numbers of sectors from left to right and top to bottom.
Figure 9: Sorted Index Betas

Note: The most dispersed patterns are obtained for smaller numbers of sectors included, i.e. the higher the number of ETFs in the portfolio the flatter the curves and vice versa.

Figure 10. Histograms of Portfolio Index Betas

Note: Increasing numbers of sectors from left to right and top to bottom
Figure 11: Sorted Coefficients of Determination

Note: Lowest and steepest patterns obtained for smaller numbers of sectors included, i.e. the higher the number of ETFs in the portfolio the higher the explained variation in their returns and vice versa.

Figure 12: Scatter Diagram of Portfolio Deadweight Scores and Portfolio Index Beta

Note: Deadweight on x-axis, beta on y-axis. Increasing numbers of sectors from left to right and top to bottom
Figure 13: Scatter Diagram of Portfolio Minbeta Scores and Portfolio Index Beta

*Note: Minbeta on x-axis, beta on y-axis. Increasing numbers of sectors left to right and top to bottom*
References


See, for example, Connelly (1997) and Gallagher and Gardner (2006)


Note that the decimals of weights have been used for calculation, rather than percentage scores, for having betas being scaled around unity rather than 100.

The provider description of the ETF is as follows: “iShares STOXX Europe 600 (DE) is an exchange traded fund (ETF) that aims to track the performance of the STOXX® Europe 600 Index as closely as possible. The ETF invests in physical index securities. The STOXX® Europe 600 Index offers exposure to the 600 largest stocks from European developed countries, measured and weighted by free float market capitalization.” For any sector, the description mirrors this accordingly with the addition being: “The STOXX® Europe 600 [Sector Name] Index offers exposure to the European [Sector Name] sector as defined by the Industry Classification Benchmark (ICB). It is a sub index of the STOXX® Europe 600 Index.”

The tracking errors for 15 out of 18 sector ETFs are less than 1% for the year under consideration, with the three sectors exceeding 1% having tracking errors of 1.1%, 1.5%, and 2.5%. For the broad index, the tracking error is 0.03%.

The computed Durbin-Watson test statistic did not indicate strong structural problems with respect to autocorrelation.