

Stochastic Calculus and Finance

Übungen

Teil 1

Problem 1

Consider a sequence of independent random variables $(\beta_n)_{n=1,2,\dots,N}$ such that β_n takes two values: $e = 2.71828 \dots$ and $-2e$ with probabilities $(2e + 1)/3e$ and $(e - 1)/3e$ correspondently. Define

$$X_n = \prod_{k=1}^n \beta_k, \quad F_n = \sigma\{\beta_1, \dots, \beta_n\}.$$

Is the sequence $(X_n, F_n)_{n=1,\dots,N}$ a martingale?

Problem 2

Let $(X_n)_{n \geq 1}$ be a sequence of independent identically distributed random variables with the density $f(x) = 1, x \in \left[-\frac{1}{2}; \frac{1}{2}\right]$ and zero outside of the segment. Define a new stochastic sequence $Y_n = \sum_{i=1}^n X_i$.

a) Is it a martingale with respect to filtration $F_n = \sigma\{X_1, \dots, X_n\}$?

How the statement is changed for densities

b) $f(x) = 1, x \in \left[-\frac{1}{3}; \frac{2}{3}\right]$ and

c) $f(x) = 1, x \in \left[-\frac{2}{3}; \frac{1}{3}\right]$ (zero outside of the segment in both cases)?